MFG model with a long-lived penalty at random jump times: application to demand side management for electricity contracts

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Renewable capacities increase worldwide

- Addition of 260 GW of renewables in 2020 (which represents 80% of all added capacities) [1].
- Almost 2800 GW of renewables worldwide (36% of total capacities), 730 GW is wind, 714 GW is solar [1].
- By the end of 2021, global renewable capacites = 3064 GW (source IRENA).

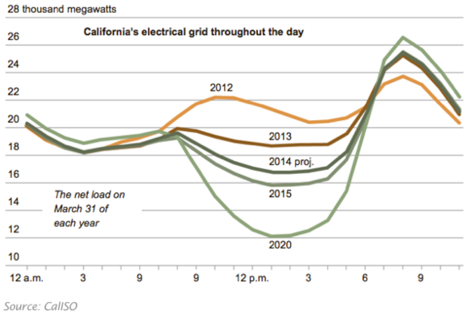


Figure: Source: CAISO

[1] IRENA, RENEWABLE CAPACITY STATISTICS 2021

Ducks do happen in reality!

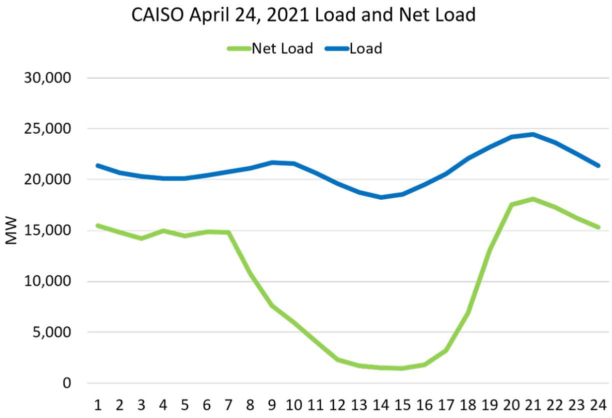


Figure: Source: CAISO

The power system requires more flexiblities.

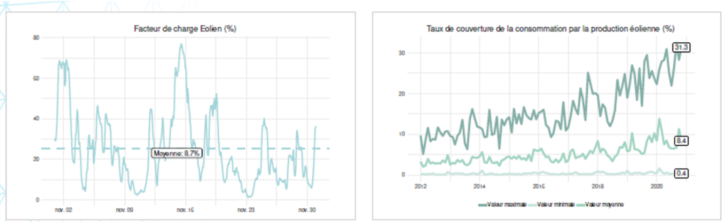


Figure: source: RTE, bilan mensuel novembre 2020

The French TSO urged all consumers to reduce their consumption on Monday 4th April morning. The spot price ended at 3000€/MWh which is the limit of the market.

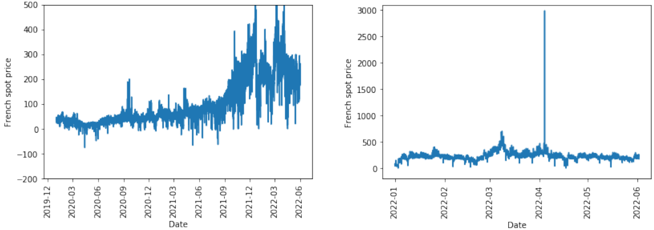
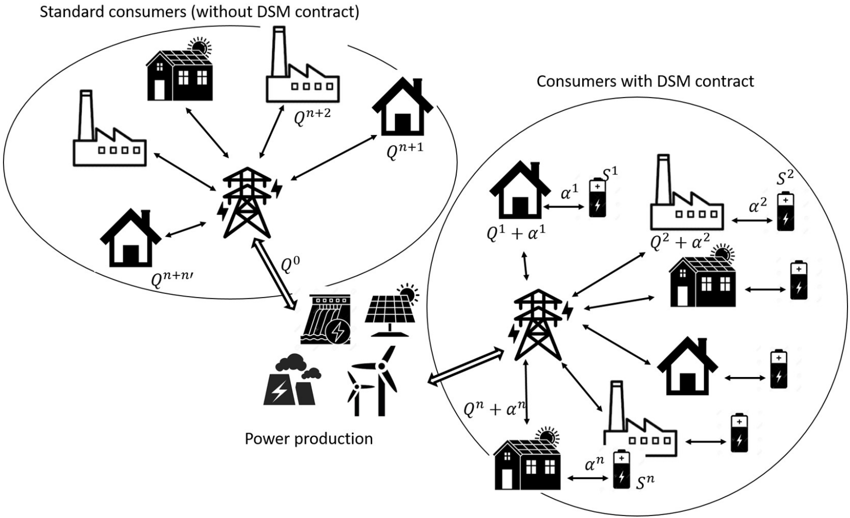


Figure: French spot price, source EpexSpot

And also **regulation incentives** to develop DSM
Clean Energy Package: each final customer should be entitled to choose a dynamic electricity price contract


Standard consumers (without DSM contract)
Q^{n+2}
Q^{n+1}
$Q^{n+n\prime}$
Q^0
Consumers with DSM contract
S^1
α^1
$Q^1 + \alpha^1$
S^2
α^2
$Q^2 + \alpha^2$
$Q^n + \alpha^n$
α^n
S^n
Power production

Objective function

We represent a DSM retail contract with two parts:

- RTP: real time pricing
- interruptible load incentive = divergence cost

Each consumer $i \in \{1, \ldots, n\}$ wants to minimise its total expected costs:

- payment of its power contract :
  - the real time tariff indexed on its energy consumption
  - the demand charge indexed on its subscribed power
  - a divergence cost when the global load does not match the interruptible load target
- inconvenience cost due to consumption modification

Spot price is sensitive to the global power demand.

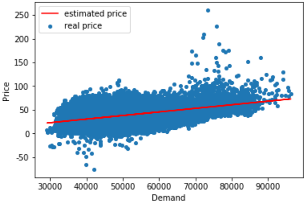


Figure: source: ENTSOE and Epexspot

The real time tariff:

$$c_t^i = (Q_t^i + \alpha_t^i) p \left(\underbrace{\frac{1}{n+n'} \sum_{j=n+1}^{n'} Q^j}_{\text{standard consumers}} + \underbrace{\frac{1}{n+n'} \sum_{j=1}^{n} (Q_t^j + \alpha_t^j)}_{\text{consumers with DSM contract}} \right).$$

or more simply $c_t^i = (Q_t^i + \alpha_t^i) p_t \left(\frac{1}{n} \sum_{j=1}^{n} (Q_t^j + \alpha_t^j) \right)$

When activated, the aim of the interruptible load contract is that the global divergence $\sum_i \alpha_t^i$ equals $\bar{\alpha}$ during θ. The divergence cost has the form:

$$d_t^i = J_t^\theta.(\tilde{Q}_t^i + \alpha_t^i - \bar{\alpha}).f\left(\frac{1}{n}\sum_{j=1}^{n}(\tilde{Q}_t^j + \alpha_t^j) - \bar{\alpha}\right)$$

- with f a convex growing function such as $f(0) = 0$
- J_t^θ equal to one during interruptible load contract activation and 0 otherwise.
- $dR_t = dt - R_{t^-}dN_t^0,\ R_0 = 2\theta,$
- $J_t^\theta = \mathbf{1}_{R_t \leq \theta}$
- $\tilde{Q}_t^i = Q_t^i - \mathbb{E}\left[Q_t^i\right]$

Each consumer $i \in \{1, \dots, n\}$ wants to minimise its total expected costs:

$$\inf_{\alpha^i \in \mathcal{A}} J_n^i(\alpha) = \inf_{\alpha^i \in \mathcal{A}} \mathbb{E}\left[\int_0^T \left(\underbrace{g(\alpha_t^i, S_t^i, Q_t^i)}_{\text{inconvenience cost}} + \underbrace{l(Q_t^i + \alpha_t^i)}_{\text{demand charge}} + \underbrace{c_t^i}_{\text{real time tariff}} + \underbrace{d_t^i}_{\text{divergence cost}} \right) dt + \underbrace{h(S_T^i)}_{\text{terminal cost}} \right],$$

with $\alpha = (\alpha^1, \dots, \alpha^n)$.

$\Longrightarrow$

- interaction of controls in real time tariff $c_t^i = (Q_t^i + \alpha_t^i) p_t \left(\frac{1}{n} \sum_{j=1}^n (Q_t^j + \alpha_t^j) \right)$
- and in divergence cost $d_t^i = J_t^\theta . (\tilde{Q}_t^i + \alpha_t^i - \bar{\alpha}) . f \left(\frac{1}{n} \sum_{j=1}^n (\tilde{Q}_t^j + \alpha_t^j) - \bar{\alpha} \right)$
- random jump time penalty: jump and delay in the divergence cost

Dynamics of the state variables

- W^0 and W two independent Brownian motions
- N^0 and N two independent Poisson processes with intensities λ^0 and λ.
- $\tilde{N}$ the compensated Poisson processes
- $\mathbb{F} = (\mathcal{F}_t)_{t\in[0,T]}$ be the (complete) natural filtration generated by (W, W^0, N, N^0, s_0, q_0).
- $\mathbb{F}^0 = (\mathcal{F}^0_t)_{t\in[0,T]}$ be the (complete) natural filtration generated by (W^0, N^0).

$$
\begin{aligned}
dQ_t &= \mu(Q_t,t)dt + \sigma(Q_t,t)dW_t + \beta(Q_{t-},t)d\tilde{N}_t + \sigma^0(Q_t,t)dW^0_t, \quad Q_0 &&= q_0,\\
dQ^{st}_t &= \mu^{st}(Q^{st}_t,t)dt + \beta(Q^{st}_{t-},t)d\tilde{N}_t + \sigma^{st}(Q^{st}_t,t)dW^0_t, \quad Q^{st}_0 &&= q^{st}_0,\\
dS_t &= \alpha_t dt, \quad S_0 &&= s_0.
\end{aligned}
$$

with $\alpha \in \mathcal{A}$, $\mathcal{A}$ the set of $\mathbb{F}$-adapted real-valued processes $a = \{a_t\}$ such that $\mathbb{E}\left[\int_0^T |a_u|^2 du\right] < \infty$ and $\mathbb{E}[|\alpha_\tau|\mathbf{1}_{\tau<\infty}] < \infty$ for all $\mathbb{F}^0$-stopping times τ with values in $[0,T] \cup \{+\infty\}$

We denote by $\tilde{Q}_t = Q_t - \mathbb{E}[Q_t]$, $t \in [0,T]$ and for a $\mathbb{F}$-adapted process $\xi = \{\xi_t\}$, denote $\hat{\xi}_t := \mathbb{E}[\xi_t|\mathcal{F}^0_t]$

MFG problem: Let $\xi = (\xi_t)_{t\in[0,T]}$ be a given $\mathbb{F}^0$-adapted process.

$$J^{MFG}(\alpha;\xi) = \mathbb{E}\left[\int_0^T \left(g(\alpha_t, S_t, Q_t) + l(Q_t+\alpha_t) + (Q_t+\alpha_t)p_t\left(\widehat{Q}_t + \xi_t\right)\right.\right.$$
$$\left.\left. + J_t^{\theta}.(\tilde{Q}_t + \alpha_t - \bar{\alpha}).f\left(\widehat{\tilde{Q}_t} + \xi_t - \bar{\alpha}\right)\right) dt + h(S_T)\right],$$

where $\alpha = (\alpha_t)_{t\in[0,T]}$ is an *admissible* control process which belongs to $\mathcal{A}$, the set of all real-valued $\mathbb{F}$-adapted processes such that $\mathbb{E}[\int_0^T \alpha_t^2 dt] < \infty$ and $\mathbb{E}[|\alpha_\tau|\mathbf{1}_{\tau<\infty}] < \infty$ for all $\mathbb{F}^0$-stopping times τ with values in $[0,T]\cup\{+\infty\}$.

$$V^{MFG}(\xi) = \inf_{\alpha\in\mathcal{A}} J^{MFG}(\alpha;\xi).$$

The goal is to find a process $\alpha^\star = (\alpha_t^\star)_{t\in[0,T]}$ such that

$$J^{MFG}(\alpha^\star;\xi) = V^{MFG}(\xi)$$

and

$$\widehat{\alpha}_t^\star = \xi_t, \text{ a.s. for all } t\in[0,T].$$

Such a process $\alpha^\star$ is called a *mean-field Nash equilibrium*.

MFG problem: Let $\xi = (\xi_t)_{t\in[0,T]}$ be a given $\mathbb{F}^0$-adapted process.

$$J^{MFG}(\alpha;\xi) = \mathbb{E}\left[\int_0^T \left(g(\alpha_t, S_t, Q_t) + l(Q_t+\alpha_t) + (Q_t+\alpha_t)p_t\left(\widehat{Q}_t + \xi_t\right)\right.\right.$$
$$\left.\left. + J_t^{\theta}.(\tilde{Q}_t + \alpha_t - \bar{\alpha}).f\left(\widehat{\tilde{Q}_t} + \xi_t - \bar{\alpha}\right)\right) dt + h(S_T)\right],$$

Hypotheses

1. $g : \mathbb{R}^3 \to \mathbb{R}$, $l : \mathbb{R} \to \mathbb{R}$ and $h : \mathbb{R} \to \mathbb{R}$ have at most quadratic growth and are strictly convex.
2. $p : \mathbb{R} \to \mathbb{R}, f : \mathbb{R} \to \mathbb{R}$ have at most linear growth.
3. g, p, f, l and h are differentiable.

Theorem (Characterization of mean field Nash equilibria)

Let $\hat{\xi}$ be a given $\mathbb{F}^0$-adapted $\mathbb{R}$-valued process and $x_0 = (s_0, q_0)$ be a random vector independent of $\mathbb{F}^0$. If there exists a control $\alpha^\star \in \mathcal{A}$ which minimizes the map $\alpha \mapsto J^{MFG}(\alpha, \hat{\xi})$ and if $(S^{\alpha^\star}, Q)$ is the state process associated to the initial condition x_0, control $\alpha^\star$ and the previous dynamics for Q and S, then there exists a unique solution $(Y^\star, q^{0,\star}, q^\star, \nu^\star, \nu^{0,\star}) \in \mathcal{S}^2 \times (\mathcal{H}^2)^4$ of the following BSDE with jumps:

$$-dY_t^\star = \partial_x g(\alpha, S_t^{\alpha^\star}, Q_t)dt - q_t^{0,\star} dW_t^0 - q_t^\star dW_t - \nu_t^\star d\widetilde{N}_t - \nu_t^{0,\star} d\widetilde{N}_t^0,$$
$$Y_T^\star = \partial_x h(S_T^{\alpha^\star}), \qquad (1)$$

satisfying the coupling condition

$$\partial_\alpha g(\alpha_t^\star, S_t^{\alpha^\star}, Q_t) + \partial_\alpha l(Q_t + \alpha_t^\star) + p_t \left(\widehat{Q}_t + \hat{\xi}_t\right) + Y_t^\star + J_t^\theta f \left(\widehat{\tilde{Q}_t} + \hat{\xi}_t - \bar{\alpha}\right) = 0. \qquad (2)$$

Conversely, assume that there exists $\left(\alpha^\star, S^{\alpha^\star}, Y^\star, q^{0,\star}, q^\star, \nu^\star, \nu^{0,\star}\right) \in \mathcal{A} \times (\mathcal{S}^2)^2 \times (\mathcal{H}^2)^4$ satisfying the coupling condition (2)*, as well as the FBSDE for S and* (1)*, then $\alpha^\star$ is the optimal control minimizing the map $\alpha \mapsto J^{MFG}(\alpha, \hat{\xi})$ and $S^{\alpha^\star}$ is the optimal trajectory.*
If additionally $\widehat{\alpha}_t^\star = \hat{\xi}_t$ a.s. for all $t \in [0, T]$, then $\alpha^\star$ is a Mean-field Nash equilibrium.

Proof: Stochastic maximum principle

MFC problem: Let π the proportion of standard consumers who do no react to price, $(1-\pi)$ the proportion of consumers who have the DSM contract:

$$
\begin{aligned}
J^C(\alpha) =& \mathbb{E}\Bigg[(1-\pi)\int_0^T \Big(g(\alpha_t, S_t, Q_t) + (Q_t+\alpha_t)p_t\left(\widehat{Q}_t+\widehat{\alpha}_t\right) \\
&\qquad + l(Q_t+\alpha_t) + J_t^\theta(\tilde{Q}_t+\alpha_t-\bar{\alpha})f\left(\widehat{\tilde{Q}_t}+\widehat{\alpha}_t-\bar{\alpha}\right)\Big)\, dt + (1-\pi)h(S_T) \\
&\qquad \pi\int_0^T\left(Q_t^{st}p_t\left(\widehat{Q}_t+\widehat{\alpha}_t\right)+l(Q_t^{st})\right)dt\Bigg].
\end{aligned}
$$

$$
V^C = \inf_{\alpha\in\mathcal{A}} J^C(\alpha). \tag{3}
$$

MFG and MFC are characterised by FBSDE systems (stochastic maximum principle) and MFC equilibrium is unique by strict convexity of the criterion.

Proposition

Consider the solution $\alpha^\star_{MFC}$ of MFC problem with a pricing rule p_{MFC} and f_{MFC}. Then $\alpha^\star_{MFC}$ is a mean field nash equilibrium for the MFG problem with pricing rule

$$p_{MFG}(x) = p_{MFC}(x) + x p'_{MFC}(x) \,,$$
$$f_{MFG}(x) = f_{MFC}(x) + x f'_{MFC}(x) \,.$$

Remark 1: Uniqueness of MFC implies the uniqueness of the MFG equilibrium.

Remark 2 : For the numerics, we use those relationships to compute the solution of the MFC by using the same code for computing both equilibria.

Semi explicit characterisation of the MFG Nash equilibrium in the linear-quadratic case

We make the following assumptions:

- $\mu(t,q) = \mu^{st}(t,q) = \mu q$ with $\mu \in \mathbb{R}$, and $\sigma(t,q) = \sigma q, \sigma^{st}(t,q) = \sigma^{st} q$, $\beta = 0$ and $\beta^{st} = 0$, with μ, σ, σ^0 given constants.
- $g(a,s,q) = \frac{A}{2}a^2 + \frac{C}{2}s^2$ with $A, C \in \mathbb{R}_+^\star$.
- $l(x) = \frac{K}{2}x^2$ with $K \in \mathbb{R}_+$.
- $f(a) = f_0 + f_1 a$ with $f_i \in \mathbb{R}$, $i = 0,1$ and $f_1 \geq 0$.
- $p(q) = p_0 + p_1 q$ with $p_0 \in \mathbb{R}$, and $p_1 \in \mathbb{R}_+^\star$.
- $h(s) = h_0 + h_1 s + \frac{h_2}{2}s^2$ with $h_i \in \mathbb{R}$, $i = 0,1,2$ and $h_2 \geq 0$.

We provide a semi explicit characterisation of the equilibrium as a decoupled system of FBSDE with jumps involving a Riccati BSDE.

We show the equilibrium approximate Nash equilibrium in the n-player game for n sufficiently large.

We propose an implementable numerical schemes.

Numerical examples - State variables based on historical Australian data

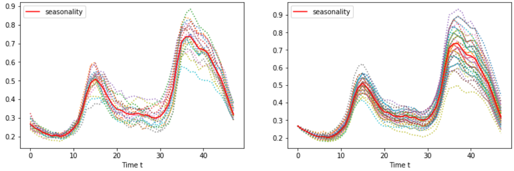


Figure: Trajectories of $\widehat{Q}$ (in kW) with estimated seasonality over 48 half-hours in a weekday in July.

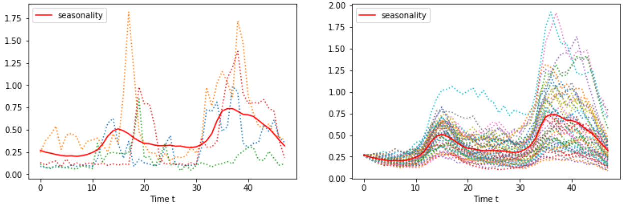


Figure: Trajectories of Q (in kW) with estimated seasonality over 48 half-hours in a weekday in July.

Scenario considered

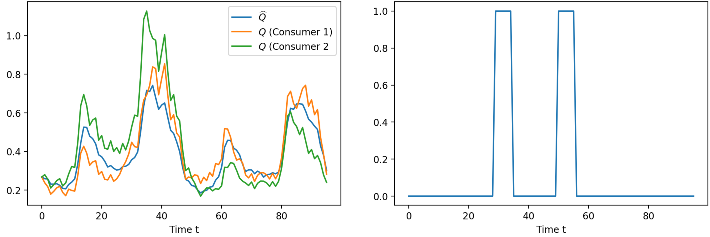


Figure: One trajectory of $\widehat{Q}$ and Q (in kW) for two different consumers (left) and one trajectory of J (right) along time (in half-hours).

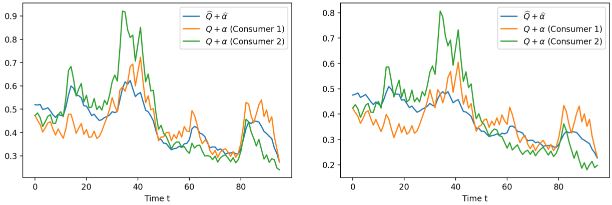


Figure: Trajectories of $\widehat{Q}+\widehat{\alpha}$ and $Q+\alpha$ (in kW) for two different consumers for MFG (left) and corresponding trajectories for MFC (right) along time (in half-hours).

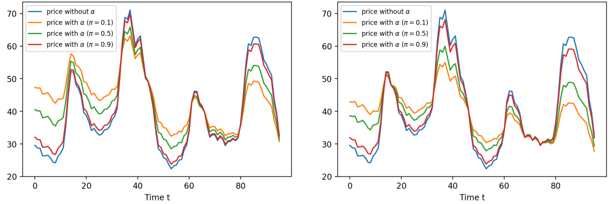
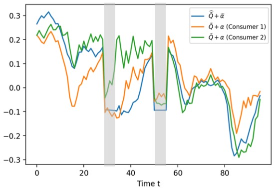


Figure: Trajectories of price p for three different proportions of active consumers for MFG (left) and corresponding trajectories for MFC (right) along time (in half-hours).



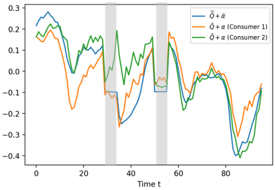


Figure: Trajectories of $\widehat{\tilde{Q}} + \widehat{\alpha}$ (in kW) and $\tilde{Q} + \alpha$ for two different consumers for MFG (left) and for MFC (right) along time (in half-hours).

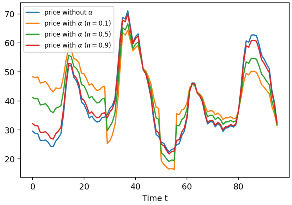


Figure: Trajectories of price p for different proportion π of standard consumers in the system in the MFG setting (jumps episodes are highlighted in grey) along time (in half-hours).

Main results:

- MFG of controls is interesting for several applications for power system with distributed local energy generation and flexibilities.
- MFG of controls with jumps and delay approach provides an analytically and numerically tractable setting to analyze the model of DSM contract.
- With quadratic cost structure and linear pricing rule, we provide quasi-explicit solutions and existence + unicity results for the equilibrium.
- A numerical implementation is proposed and provides interesting results.
- Centralised optimization can be decentralized: extended MFG can linked to suitable Mean Field Type Control (MFC) problem (central planner point of view)

Perspectives:

- Study a Stackelberg game: add an aggregator who designs the DSM contrat
- Optimise the activation of the interruptible load J^{θ}.
- Achieve numerics with more general settings

- Electrical system and MFG:
  R. Couillet, R., S. Medina Perlaza, H. Tembine, H. and M. Debbah (2012), D. Bauso (2017), A. de Paola, D. Angeli, and G. Strbac, G. (2016 et 2019), D. Gomes, J. Saude (2018), A. De Paola, V. Trovato, D. Angeli, G. Strbac (2019), C.A, I. Ben Tahar, and A. Matoussi (2020).
- Extended MFG and MFG with common noise : R. Carmona and F. Delarue (2013), (2015), (2017); R. Carmona and F. Delarue and D. Lacker (2016), D. Gomes & al. (2013), (2016), P. Cardaliaguet and C.A. Lehalle (2017)
- Games with delay: R. Carmona, JP. Fouque, SM. Mousavi, LH. Sun(2018), Bensoussan & al. (2016)
- MFG with jumps: C. Benazzoli, L. Campi, and L. Di Persio (2019 and 2020), Z. Li, A. M. Reppen, and R. Sircar (2019)
- Linear Quadratic case: A. Bensoussan, Yong (2013), Pham (2016), Graber (2016), Sun(2015)
- Numerics: . Lejay, E. Mordecki, and S. Torres (2014), R. Dumitrescu and C. Labart (2016)