Switching problems with controlled randomisation and associated obliquely reflected BSDEs

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Annecy

Outline

– Switching with controlled randomisation: examples, mathematical formulation, associated BSDE and verification theorem

– Randomised switching with signed costs: study of the geometry of the domain and an existence theorem.

Classical switching problems

– Classical litterature: Hamadène and Jeanblanc (2005, two modes), Djehiche, Hamadène and Popier (2007, d modes), Hu and Tang (2010, controlled drift, switched BSDEs, driver $f^i(t, y^i, z^i)$), Elie, Kharroubi (2011, controlled volatility), Chassagneux, Elie, Kharroubi (2012, driver $f^i(t, y, z^i)$).

– Notations: Time horizon $0 < T < \infty$. Probability space $(\Omega, \mathcal{G}, \mathbb{P})$, Brownian motion W, $\mathbb{F}^0 = (\mathcal{F}^0_t)_{t\in[0,T]}$ its augmented filtration.
– Control problem, starting from mode $i \in \{1, \ldots, d\}$ at time $t \in [0, T]$,

$$\mathcal{V}^i_t = \operatorname{ess}\sup_{(\tau_n,\zeta_n)_{n\geq 1}} \mathbb{E}\left[\int_t^T \psi_{a_s}(X_s)\mathrm{d}s + g_{a_T}(X_T) - \sum_{n\geq 1} c_{\zeta_{n-1},\zeta_n} 1_{\{\tau_n<T\}} \middle| \mathcal{F}^0_t\right],$$

where X is an underlying stochastic process.
– A strategy is $(\tau_n, \zeta_n)_{n\geq 0}$ where (τ_n) is a sequence of stopping times (switching times) and ζ_n is the mode on $[\tau_n, \tau_{n+1})$.
– State process: $a_t = \sum_{n\geq 0} \zeta_n 1_{\tau_n\leq t<\tau_{n+1}}$, $t \in [0, T]$.
– Admissibility: strategy with $\mathbb{E}\left[\left(\sum_{n\geq 0} 1_{\tau_n\leq T}\right)^2\right] < \infty$.

– Process $\mathcal{V}$ lives in a convex domain of $\mathbb{R}^d$ and solves a BSDE with oblique reflections.

Switching with controlled randomisation

Switching with (controlled) randomisation

– The agent do not directly choose the new mode when she decides to switch.

– Randomised switching: the new mode is decided randomly (independently of everything up to now), according to a (known) distribution on $\{1, \dots, d\}$.
$\hookrightarrow$ strategy $(\tau_n)_{n\geq 1}$ nondecreasing sequence of random times.
If actual mode is i and the agent decides to switch, she pays cost $\bar{c}_i$.

– Controlled randomisation: the agent first chooses a distribution in $\{P^u : u \in \mathcal{C}\}$. The new mode is drawn according to this distribution.
$\hookrightarrow$ strategy $(\tau_n, \alpha_n)_{n\geq 1}$ where α_n is the chosen distribution at time τ_n.
If actual mode is i and the agent decides to switch using law $P^u, u \in \mathcal{C}$, the cost is $\bar{c}_i^u$.

– Remark: randomised switching is a particular case of controlled randomisation when $\mathcal{C}$ is a singleton.

Example – randomised switching

– Assume $d = 3$.

– Here the agent only decides to switch, do not control the distribution of the new mode.

– New mode decided independently with transition matrix and cost

$$P = \begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \end{pmatrix}, \bar{c} = \begin{pmatrix} 0.5 \\ 0.5 \\ 0.5 \end{pmatrix}.$$

$\hookrightarrow$ When the agent wants to switch, the new mode is determined by throwing a fair coin.

For example, if the present mode is 1 and the agent wants to switch, the new mode is 2 with probability 0.5 and 3 with probability 0.5.

Example – switching with controlled randomisation

– Here the agent decides when to switch and chooses the distribution of the new mode.

She chooses $u \in [0,1]$, and the new mode is determined by

$$P^u = \begin{pmatrix} 0 & u & 1-u \\ 1-u & 0 & u \\ u & 1-u & 0 \end{pmatrix}, \bar{c}^u = \begin{pmatrix} 1-u(1-u) \\ 1-u(1-u) \\ 1-u(1-u) \end{pmatrix}.$$

– Example: current mode is 1 and switching with control u
$\hookrightarrow$ new mode is 2 (resp. 3) with probability u (resp. $1-u$).

$\hookrightarrow$ To increase the probability to be in mode 2 after the switch, the agent should take u closer to 1.

$\hookrightarrow$ Reducing uncertainty induces a higher cost as $\bar{c}^u_1$ is higher with u closer to 1.

– Applications to risk aversion.

Classical switching

– For each $d \geq 2$, the classical switching problem is a particular case of switching with controlled randomisation.

– For $d = 3$ for example, the transition matrices are:

$$P^1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \bar{c}^1 = \begin{pmatrix} c_{1,2} \\ c_{2,3} \\ c_{3,1} \end{pmatrix},$$

$$P^2 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \bar{c}^2 = \begin{pmatrix} c_{1,3} \\ c_{2,1} \\ c_{3,2} \end{pmatrix}.$$

Some comments

– When the agent decides to switch, the new mode is chosen with some extra and independent noise $\hookrightarrow$ mathematical analysis must deal with enlargement of filtrations.

– The enlarged filtration depends on the switching times, hence on the control.

– Classical switching $\hookrightarrow$ "triangular inequality" $c_{i,j} + c_{j,k} > c_{i,k}$ $\hookrightarrow$ no simultaneous switches.
Here, in general, the question of simultaneous switches arises: the agent may not be satisfied with the randomly reached state.

Setup

– Control set: $\mathcal{C}$ an ordered compact metric space.

– Probability space: $(\Omega, \mathcal{G}, \mathbb{P})$ with $\mathcal{G} = \sigma(W, (\mathfrak{U}_n)_{n\geq 1})$.
W is a κ-dimensional Brownian motion and $\mathbb{F}^0$ its augmented natural filtration.
$(\mathfrak{U}_n)_{n\geq 1}$ i.i.d. family of uniform r.v.'s on $[0,1]$, independent of W $\hookrightarrow$ models extra-randomness at switching times.

– Switching: if present mode is $i \in \{1, \dots, d\}$ and agent wants to switch with control $u \in \mathcal{C}$ $\hookrightarrow$ new mode $F(u, i, \mathfrak{U}) \in \{1, \dots, d\}$ with $\mathfrak{U}$ uniform on $[0,1]$ and cost $\bar{c}_i^u$, where $\bar{c} : \{1, \dots, d\} \times \mathcal{C} \to \mathbb{R}$ is continuous.
We set $P_{i,j}^u = \mathbb{P}(F(u, i, \mathfrak{U}) = j)$.

– Data: (similar to Hu and Tang (2010))
terminal condition $\hookrightarrow \xi = (\xi^1, \dots, \xi^d) \in L^2(\mathcal{F}_T^0)$,
driver $\hookrightarrow f : \Omega \times [0,T] \times \mathbb{R}^d \times \mathbb{R}^{d\times\kappa} \to \mathbb{R}^d$ satisfying to

- $f(\cdot, 0, 0) \in \mathbb{H}^2(\mathbb{F}^0)$ and f is progressive,
- For all $(t, y, z) \in [0,T] \times \mathbb{R}^d \times \mathbb{R}^{d\times\kappa}$, $f^i(t, y, z) = f^i(t, y_i, z_i)$.
- For all $(t, y^1, y^2, z^1, z^2) \in [0,T] \times (\mathbb{R}^d)^2 \times (\mathbb{R}^{d\times\kappa})^2$,

$$|f(t, y^1, z^1) - f(t, y^2, z^2)| \leq L(|y^1 - y^2| + |z^1 - z^2|).$$

Strategies

– Strategy for the problem starting at t in i:
$\phi = (\zeta_0 = i, \tau_0 = t, (\tau_n, \alpha_n)_{n\geq 1})$ where

- $(\tau_n, \alpha_n)_{n\geq 1}$ is a sequence of $\mathcal{G}$-random variables valued in $[t, \infty) \times \mathcal{C}$,
- $\tau_n \leq \tau_{n+1}$ for all $n \geq 0$, and
- for $n \geq 0$, τ_{n+1} is a $\mathbb{F}^n$-stopping time and α_{n+1} is $\mathcal{F}^n_{\tau_{n+1}}$-measurable. We then set $\mathbb{F}^{n+1} = (\mathcal{F}^{n+1}_t)_{t\geq 0}$ with $\mathcal{F}^{n+1}_t = \mathcal{F}^n_t \vee \sigma(\mathfrak{U}_{n+1} 1_{\{\tau_{n+1} \leq t\}})$ for all $t \geq 0$.

– For all $n \geq 0$ and $s \in [t, T]$, we define:

- the state after $n+1$ switches as $\zeta_{n+1} = F(\alpha_{n+1}, \zeta_n, \mathfrak{U}_{n+1})$,
- the state process as $a_s = \sum_{n\geq 0} \zeta_n 1_{[\tau_n, \tau_{n+1})}(s)$ and
- the cumulative cost process as $A^\phi_s = \sum_{n\geq 0} \bar{c}^{\alpha_{n+1}}_{\zeta_n} 1_{\tau_{n+1} \leq s}$.

– We define the filtration associated to the strategy as $\mathbb{F}^\infty = (\mathcal{F}^\infty_t)_{t\geq 0}$ with $\mathcal{F}^\infty_t = \bigvee_{n\geq 0} F^n_t$, $t \geq 0$.

– A strategy ϕ is admissible ($\phi \in \mathcal{A}^i_t$) if

$$A^\phi_T - A^\phi_t \in L^2(\mathcal{F}^\infty_T) \text{ and } \mathbb{E}\left[(A^\phi_t)^2 | \mathcal{F}^0_t\right] < +\infty.$$

Switching problem with controlled randomisation

– Given an admissible strategy ϕ, the associated reward is given by (see Hu and Tang (2010)):

$$\mathbb{E}\left[U_t^\phi - A_t^\phi | \mathcal{F}_t^0\right],$$

with (U^ϕ, V^ϕ, M^ϕ) being the solution in $\mathbb{F}^\infty$ to the following **switched BSDE**: for $s \in [t, T]$,

$$U_s = \xi^{a_T} + \int_s^T f^{a_r}(r, U_r, V_r)\mathrm{d}r - \int_s^T V_r \mathrm{d}W_r - \int_s^T \mathrm{d}M_r - \int_s^T \mathrm{d}A_r^\phi,$$

– **Proposition:** For ϕ admissible, $\mathbb{F}^\infty$ is right-continuous and there exists a unique solution to the BSDE.

– M^ϕ is a $\mathbb{F}^\infty$-martingale $\hookrightarrow$ we obtain a martingale representation theorem: M^ϕ jumps only at the switching times of the strategy associated to $\mathbb{F}^\infty$.

– Problem value, starting in mode $i \in \{1, \dots, d\}$ at time $t \in [0, T]$:

$$\mathcal{V}_t^i = \operatorname*{ess\,sup}_{\phi \in \mathcal{A}_t^i} \mathbb{E}\left[U_t^\phi - A_t^\phi | \mathcal{F}_t^0\right].$$

Particular case

– Assume the driver does not depend upon U, V: $f(\omega, t, u, v) = f(\omega, t)$.

– Then, for ϕ admissible,

$$\mathbb{E}\left[U^{\phi} - A_t^{\phi} \,\middle|\, \mathcal{F}_t^0\right] = \mathbb{E}\left[\xi^{a_T} + \int_t^T f^{a_s}(s)\mathrm{d}s - A_T^{\phi} \,\middle|\, \mathcal{F}_t^0\right]$$
$$= \mathbb{E}\left[\xi^{a_T} + \int_t^T f^{a_s}(s)\mathrm{d}s - \sum_{n\geq 0} \bar{c}_{\zeta_n}^{\alpha_{n+1}} 1_{\{\tau_{n+1}\leq T\}} \,\middle|\, \mathcal{F}_t^0\right].$$

– The problem thus writes:

$$\mathcal{V}_t^i = \operatorname*{ess\,sup}_{\phi\in\mathcal{A}_t^i} \mathbb{E}\left[\xi^{a_T} + \int_t^T f^{a_s}(s)\mathrm{d}s - \sum_{n\geq 0} \bar{c}_{\zeta_n}^{\alpha_{n+1}} 1_{\{\tau_{n+1}\leq T\}} \,\middle|\, \mathcal{F}_t^0\right].$$

The domain of reflections

– Classical switching: value $\mathcal{V}$ linked to the solution of an obliquely reflected BSDE in some convex domain. Similar here with positive costs.

– Heuristically, the maximal profit is greater than the expected profit obtained by the strategy:

1. Switching instanteanously with control $u \in \mathcal{C}$, leading to mode j with probability $P^u_{i,j}$.
2. Following the optimal strategy in the new mode.

Then $\mathcal{V}^i_t \geq \mathbb{E}[\mathcal{V}^\zeta_t] - \bar{c}^u_i$ with ζ the (random) mode after switching from i with control u.

– Since this strategy is available for each $u \in \mathcal{C}$, we obtain

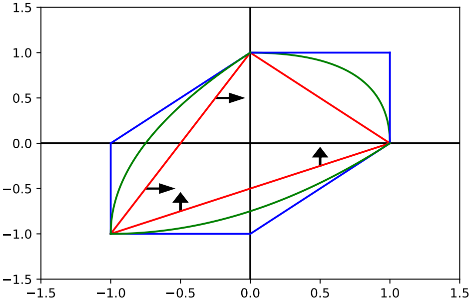
$$\mathcal{V}^i_t \geq \sup_{u \in \mathcal{C}} \left(\sum_{j=1}^{d} P^u_{i,j} \mathcal{V}^j_t - \bar{c}^u_i \right) .$$

– The problem value lies into the following convex domain of $\mathbb{R}^d$:

$$\mathcal{D} = \left\{ y \in \mathbb{R}^d \,\middle|\, y_i \geq \sup_{u \in \mathcal{C}} \left(\sum_{j=1}^{d} P^u_{i,j} y_j - \bar{c}^u_i \right), 1 \leq i \leq d \right\} .$$

Examples of domains

– Easy lemma: let $\mathcal{D}_0 := \{y \in \mathcal{D} \,|\, y_d = 0\}$. Then $\mathcal{D} = \mathcal{D}_0 \oplus \mathbb{R} \cdot (1, \ldots, 1)$.
$\hookrightarrow$ $\mathcal{D}$ is obtained by translating $\mathcal{D}_0$ along the axis $\mathbb{R} \cdot (1, \ldots, 1)$.



Graphs of $\mathcal{D}_0 = \mathcal{D} \cap \{y_3 = 0\}$ as a subset of $\{(y_1, y_2, 0)\} \simeq \mathbb{R}^2$.
Blue: the usual switching domain with cost 1,
Red: domain from example 1,
Green: domain from example 2.

The BSDE

- Heuristically, as in the classical case, one expects that $\mathcal{V} = Y$, where (Y, Z, K) is the solution to the following *Obliquely Reflected BSDE*

$$Y_t^i = \xi^i + \int_t^T f^i(s, Y_s^i, Z_s^i)\mathrm{d}s - \int_t^T Z_s^i \mathrm{d}W_s + \int_t^T \mathrm{d}K_s^i, \tag{1}$$

$$Y_t \in \mathcal{D} \text{ and } \int_0^T \left(Y_t^i - \sup_{u \in \mathcal{C}} \left(\sum_{j=1}^d P_{i,j}^u Y_t^j - \bar{c}_i^u \right) \right) \mathrm{d}K_t^i = 0, \tag{2}$$

and that an optimal strategy starting at $t = \tau_0 \in [0, T]$ and mode $\zeta_0 \in \{1, \dots, d\}$ is given by

$$\tau_{k+1}^\star = \inf \left\{ s \geq \tau_k^\star \;\middle|\; Y_s^{\zeta_k^\star} = \sup_{u \in \mathcal{C}} \left(\sum_{j=1}^d P_{\zeta_k^\star, j}^u Y_s^j - \bar{c}_{\zeta_k^\star}^u \right) \right\} \wedge (T+1),$$

$$\alpha_{k+1}^\star = \inf \operatorname*{argsup}_{u \in \mathcal{C}} \left(\sum_{j=1}^d p_{\zeta_k^\star, j}^u Y_s^j - \bar{c}_{\zeta_k^\star}^u \right).$$

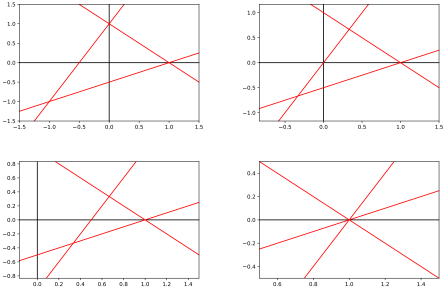
– This is indeed true in a positive costs setting.

– One easily deduces **uniqueness of solutions** in a signed costs setting.

Randomised switching with signed costs

Study of the domain of reflection

– We now assume randomisd switching: $\mathcal{C} = \{0\}$, i.e. the agent do not control the distribution of the new state.
– We set $P = (P_{i,j})_{i,j}$ and $\bar{c} = (\bar{c}_i)_i \in \mathbb{R}^d$ (signed costs).
– A first issue is that it is not *a priori* guaranteed that the domain $\mathcal{D}$ has non-empty interior, or at least is non-empty.



Domain $\mathcal{D}_0 = \mathcal{D} \cap \{y_3 = 0\}$, for $\bar{c}_1 \in \{0.5, 0, -0.5, -1\}$ in the example of randomised switching.
For $\bar{c}_1 = -1$, the domain has empty interior and for $\bar{c}_1 < -1$ the domain is empty!

Study of the domain of reflection

– We assume that P is irreducible, and we let μ be its unique invariant probability measure.

– In this setting, the domain is

$$\mathcal{D} = \left\{ y \in \mathbb{R}^d : y \succcurlyeq Py - \bar{c} \right\},$$

with $\succcurlyeq$ the component by component partial ordering.

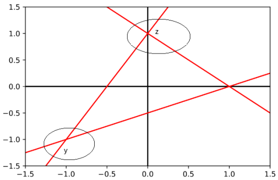
– If $y \in \mathcal{D}$, we have $\mu y \geq \mu P y - \mu\bar{c} = \mu y - \mu\bar{c}$, hence $\mu\bar{c} \geq 0$.

– Questions: Conversely, if $\mu\bar{c} \geq 0$, can we conclude that $\mathcal{D}$ is non-empty? What about the condition $\mu\bar{c} > 0$? How to interpret the condition $\mu\bar{c} \geq 0$ in terms of the switching problem?

– Recall that $\mathcal{D} = \mathcal{D}_0 \oplus \mathbb{R} \cdot (1, \ldots, 1)$
$\hookrightarrow$ $\mathcal{D}$ is non-empty (resp. has non-empty interior) iif $\mathcal{D}_0$ is (resp. has non-empty interior) in $\{y_d = 0\} \simeq \mathbb{R}^{d-1}$.

– Randomised switching with irreducible transition matrix
$\hookrightarrow$ $\mathcal{D}_0$ is a simplex.
$\hookrightarrow$ what are the coordinates of its vertices?

Study of the domain of reflection



– If $\mathcal{V}_t = y$, constraints $y_1 \geq Py - \bar{c}_1$ and $y_2 \geq Py - \bar{c}_2$ are both saturated, i.e. if the current mode is 1 or 2, it is optimal to switch.
$\hookrightarrow$ optimally, if current mode is 1 (or 2), simultaneous switches are needed until mode 3 is reached. Then apply optimal strategy from mode 3 for optimal reward $\mathcal{V}_t^3 = y_3 = 0$.
$\hookrightarrow y_1 = \mathcal{V}_t^1 = \mathcal{V}_t^3 - C_{1,3} = -C_{1,3}$ with $C_{i,j} =$ mean cost to reach j from i.
$\hookrightarrow y = (-C_{1,3}, -C_{2,3}, 0)$.

– Similar argument can be applied to $z = (z_1, z_2, 0)$: optimal to switch to mode 2, where optimal reward is z_2. Thus $z = (z_2 - C_{1,2}, z_2, z_2 - C_{3,2})$, and since $z_3 = 0$, one gets $z_2 = C_{3,2}$ and $z = (C_{3,2} - C_{1,2}, C_{3,2}, 0)$.

Study of the domain of reflection

– For $(i,j) \in \{1,\ldots,d\}^2$, the key quantity is the expected cost along an excursion from state i to state j:

$$C_{i,j} = \mathbb{E}\left[\sum_{n=0}^{\tau_j - 1} \bar{c}_{X_n} \,\middle|\, X_0 = i\right],$$

where X is the irreducible Markov chain with transition matrix P and $\tau_j = \inf\{n \geq 0 | X_n = j\}$.

– More technical $\hookrightarrow$ Combining linear algebra and Markov Chain arguments, link between $\mu\bar{c}$ and the $C_{i,j}$'s.

Theorem

The following conditions are equivalent:

1. The domain $\mathcal{D}$ is non-empty (resp. has non-empty interior).
2. There exists $1 \leq i \neq j \leq d$ such that $C_{i,j} + C_{j,i} \geq 0$ (resp. $C_{i,j} + C_{j,i} > 0$).
3. The inequality $\mu\bar{c} \geq 0$ is satisfied (resp. $\mu\bar{c} > 0$)
4. For all $1 \leq i \neq j \leq d$, we have $C_{i,j} + C_{j,i} \geq 0$ (resp. $C_{i,j} + C_{j,i} > 0$).

Study of the domain of reflection

– We recover the triangular inequality with the $C_{i,j}$'s:

Corollary

The following conditions are equivalent:

1. The domain $\mathcal{D}$ is non-empty.
2. For all $1 \leq i, j, k \leq d$, we have $C_{j,k} \leq C_{j,i} + C_{i,k}$.
3. For any round trip of length less that d, i.e. $1 \leq n \leq d$ and $1 \leq i_1 \neq \ldots \neq i_n \leq d$, we have $\sum_{k=1}^{n-1} C_{i_k, i_{k+1}} + C_{i_n, i_1} \geq 0$.

– Remark: In the case of classical switching problems, this triangular inequality is satisfied with the costs to switch from mode i to mode j. Here, with randomised switching, we need to consider the expected cost to switch from mode i to j.

Existence of solutions to the BSDE

– Chassagneux and Richou (2020): studied obliquely reflected BSDEs in general.

$\hookrightarrow$ They study solutions (Y, Z, K) to obliquely BSDEs of the form

$$Y_t = \xi + \int_t^T f(s, Y_s, Z_s)\mathrm{d}s - \int_t^T Z_s \mathrm{d}W_s - \int_t^T H(s, Y_s, Z_s)\Phi_s \mathrm{d}s,$$

$$Y \in \mathcal{D}, \Phi \in n_{\mathcal{D}}(Y), \int_0^T |\Phi_t| 1_{\{Y_t \notin \partial\mathcal{D}\}}\mathrm{d}t = 0,$$

where

- $n_{\mathcal{D}}(y)$ is the outward normal cone at Y for the convex domain $\mathcal{D}$,
- $H \in \mathbb{R}^{d\times d}$ is a given operator allowing for oblique reflections satisfying to technical assumptions.

– Our task: construct an operator H such that $H(y)n_{\mathcal{D}}(y) \subset C_o(y)$ the oblique cone for y for our problem, and check that H mets the technical assumptions to apply the existence results.

$\hookrightarrow$ compute the cones at each $y \in \partial\mathcal{D}_0$, define $H = H(y)$ first on $\partial\mathcal{D}_0$, then extend it to $\mathcal{D}_0$ by convexity, to $\mathcal{D}$ and finally to $\mathbb{R}^d$ by projection.

Existence of solutions to the BSDE

– Notations: $Q := I_d - P$ and for each $1 \leq i \leq d$, we set $Q^{(i,i)}$ the square matrix of size $d-1$ obtained from Q by deleting row i and column i.

– Markovian framework: $\xi = g(X_T^{t,x})$ and $f(\omega, s, y, z) = \psi(s, X_s^{t,x}(\omega), y, z)$ for some maps $g : \mathbb{R}^q \to \mathbb{R}^d$ and $\psi : [0, T] \times \mathbb{R}^q \times \mathbb{R}^d \times \mathbb{R}^{d \times \kappa}$ and X a Itô diffusion.

Theorem

Assume some technical conditions on the maps g, f and the coefficients b and σ of the dynamics of X.
Assume $\mathcal{D}$ has non-empty interior.
Moreover, assume that for all $1 \leq i \leq d$, the matrix $Q^{(i,i)}$ satisfies the following **copositivity hypothesis**: for all $\mathbb{R}^{d-1} \ni x \succcurlyeq 0$, $x \neq 0$, we have

$$x^\top Q^{(i,i)} x > 0.$$

Then

- we can construct a H satisfying to the technical assumptions,
- the reflected BSDE (1)-(2) admits a solution.

Remarks

– The copositivity hypothesis is always satisfied when $d = 3$.

– In dimension $d \geq 3$, this hypothesis is satisfied for the randomised switching with transition matrix $P_{i,j} = \frac{1}{d-1} 1_{i \neq j}$.

– A counter-example in dimension $d = 4$:

$$P = \begin{pmatrix} 0 & \frac{\sqrt{3}}{2} & 0 & 1 - \frac{\sqrt{3}}{2} \\ 1 - \frac{\sqrt{3}}{2} & 0 & \sqrt{3} - 1 & 1 - \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}.$$

Conclusion and further work

– We defined a new switching problem with uncertainty on the new mode when the agent decides to switch.
– When the costs are positive, we obtained a representation theorem in terms of a BSDE with oblique reflection, which implies the uniqueness for the BSDE.
– When the costs are signed and in the setting of randomised switching, we obtain a characterisation of the non-emptiness (using the control problem data) for the domain of reflections.
– We obtain existence in a Markovian framework for the randomised switching problem. In the paper, we have examples of existence for a controlled randomisation, and in a non-Markovian framework.

– The general study of existence of the BSDEs associated to switching problems with controlled randomisation remains **open**.
– A representation theorem with signed costs is **not** proved.
– Extension to **time-dependent** and **random** costs and transition probabilities.

Thank you for your attention!