BSDEs with Jumps in the L^p-setting: Existence, Uniqueness and Comparison

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joint work with S. Kremsner, (University of Graz, s2-data)

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For Lévy-processes X: We consider

$$Y_t = \xi + \int_t^T f(s, Y_s, Z_s, U_s)ds - \int_t^T Z_s dW_s - \int_{]t,T]\times\mathbb{R}_0^d} U_s(x)\tilde{N}(ds, dx),$$
$$0 \leq t \leq T,$$

X has Lévy-Itô decomposition

$$X_t = \gamma t + \sigma W_t + \int_{(0,t]\times\{|x|<1\}} x\widetilde{N}(ds, dx) + \int_{(0,t]\times\{|x|\geq 1\}} xN(ds, dx)$$

and Lévy-measure ν.

$N, \widetilde{N}$...(compensated) Poisson random measure.

Assumptions for the BSDE

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- $\xi \in L^p$, $p > 1$
- searching for solutions (Y, Z, U) in $S^p \times L^p(W) \times L^p(\tilde{N})$, where
  - $S^p := \left\{ Y : Y \text{ prog.-mble. and } \mathbb{E} \sup_{t\in[0,T]} |Y_t|^p < \infty \right\}$
  - $L^p(W) := \left\{ Z : Z \text{ prog.-mble. and } \mathbb{E} \left(\int_0^T |Z_t|^2 dt \right)^{\frac{p}{2}} < \infty \right\}$
  - $L^p(\tilde{N}) := \left\{ U : U \text{ prog.-mble. and } \mathbb{E} \left(\int_0^T \int_{\mathbb{R}_0^d} |U_t(x)|^2 dt \right)^{\frac{p}{2}} < \infty \right\}$

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- such that $(t, \omega) \mapsto f(\omega, t, y, z, u)$ is progressively measurable for all (y, z, u).
- $(y, z, u) \mapsto f(\omega, t, y, z, u)$ is continuous $\mathbb{P}$-a.s.

Growth condition in y:

$$Y_t = \xi + \int_t^T f(s, Y_s, Z_s, U_s)ds - \int_t^T Z_s dW_s - \int_{]t,T]\times\mathbb{R}_0^d} U_s(x)\tilde{N}(ds, dx),$$
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- For all $r > 0$, ψ_r is nonnegative, progressively measurable and $\mathbb{E}\int_0^T \psi_r(s)ds < \infty$.

Extended monotonicity condition, $p \geq 2$:

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- $|y - y'|^{p-2}\langle y - y', f(t, y, z, u) - f(t, y', z', u')\rangle \leq \alpha(t)\rho(|y - y'|^2)|y - y'|^{p-2} + \mu(t)|y - y'|^p + \beta(t)|y - y'|^{p-1}(|z - z'| + \|u - u'\|_\nu)$

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- μ, β nonnegative, progressively measurable and $\int_0^T (\mu(s) + \beta(s)^2)\, ds < c$, $\mathbb{P}$-a.s.

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- $|y - y'|^{p-2}\langle y - y', f(t, y, z, u) - f(t, y', z', u')\rangle \leq$
  $\alpha(t)\rho(|y - y'|^2)|y - y'|^{p-2} + \mu(t)|y - y'|^p$
  $+\beta(t)|y - y'|^{p-1}(|z - z'| + \|u - u'\|_\nu)$
- ρ: $[0, \infty[\to [0, \infty[$ is concave, nondecreasing, $\rho(0) = 0$,
  $\int_{0+} \frac{1}{\rho(x)} dx = \infty$ and $\lim_{x\searrow 0} \frac{\rho(x^2)}{x} = 0$

Extended monotonicity condition, $1 < p < 2$::

$$Y_t = \xi + \int_t^T f(s, Y_s, Z_s, U_s)ds - \int_t^T Z_s dW_s - \int_{]t,T]\times\mathbb{R}_0^d} U_s(x)\tilde{N}(ds, dx),$$
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- $|y - y'|^{p-2}\langle y - y', f(t, y, z, u) - f(t, y', z', u')\rangle \le$
  $\alpha(t)\rho(|y - y'|^p) + \mu(t)|y - y'|^p$
  $+|y - y'|^{p-1}(\beta_1(t)|z - z'| + \beta_2(t)\|u - u'\|_\nu)$
- $\alpha \in L^1([0, T])$
- $\int_0^T (\beta_1(t)^2 + \beta_2(s)^q)ds < c$, $\mathbb{P}$-a.s. for some $q > 2$.

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- The function ρ, extending the monotonicity to an Osgood condition allows to extend standard techniques which normally use Gronwall's inequality through the Bihari-LaSalle-inequality.
- We avoid the definition of specially designed spaces, just keeping the $S^p \times L^p(W) \times L^p(\tilde{N})$ also for the case $1 < p < 2$.

Existence theorem in the present case:

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Theorem

If $\xi \in L^p$ and $p > 1$, and the mentioned assumptions hold for f, then there exists a unique solution to the given BSDE s.t.
$\mathbb{E} \sup_t |Y_t|^p + \mathbb{E} \left[\int_0^T (|Z_t|^2 + \|U_t\|_\nu^2) dt \right]^{\frac{p}{2}} < \infty.$

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A priori estimate:

Theorem

Let $(\xi, f), (\xi', f')$ satisfy the above assumptions. Then

$$\sup_t \mathbb{E}|Y_t - Y_t'|^p + \mathbb{E}\left[\int_0^T |Z_t - Z_t'|^2 ds\right]^{\frac{p}{2}} + \mathbb{E}\left[\int_0^T \|U_t - U_t'\|_\nu^2 dt\right]^{\frac{p}{2}}$$
$$\leq h\left(\|\xi - \xi'\|_{L^p}, \left\|\int_0^T |f(t, Y_t, Z_t, U_t) - f'(t, Y_t, Z_t, U_t)|dt\right\|_{L^p}\right).$$

$h(u, v) \to 0$ *if* $(u, v) \to (0, 0)$.

Similar settings have been considered e.g. in:
Kruse and Popier '16, '17
Yao '17
C. Geiss and S. '18

	growth in y	p	activity	coefficients
Kruse & Popier	general	$p \geq 2$ OK, $1 < p < 2$ in special spaces	infinite	constant
Yao	general	$1 < p < 2$	finite	ρ, time-dep.
G.& S.	linear	$p = 2$ only	infinite	ρ, time-dep.
K.& S.	general	$p > 1$	infinite	ρ, time-dep.

Comparison theorems for BSDEs with jumps need an additional condition (counterexample in Barles et al. '97).

Standard conditions: $\xi \leq \xi'$,

$$f(Y, Z, U) \leq f'(Y, Z, U) \quad \text{or} \quad f(Y', Z', U') \leq f'(Y', Z', U')$$

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Additional condition: f or f' satisfy (A γ)

$$f(t, y, z, u) - f(t, y, z, u') \leq \int_{\mathbb{R}\setminus\{0\}} (u'(x) - u(x))\nu(dx),$$

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Then, $Y_t \leq Y'_t$, $\mathbb{P}$*-a.s.*

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  (C. Geiss & S. $\rightarrow$ linear growth in y)
- Now: 'Cut off' the Lévy measure directly.

Back to the Lévy-Itô decomposition:

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- $\mathbb{E}^n = \mathbb{E}[\,\cdot\,|\mathcal{F}^n]$.

Cut-off-BSDE:

$$\mathbb{E}^n Y_t = \mathbb{E}^n \xi + \int_t^T \mathbb{E}^n f(s, Y_s, Z_s, U_s) ds - \int_t^T \mathbb{E}^n Z_s dW_s - \int_{]t,T] \times \{|x| < \frac{1}{n}\}} \mathbb{E}^n U_s(x) \tilde{N}(ds, dx), \quad 0 \le t \le T,$$

- $\mathbb{E}^n Y_t \to Y_t$
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- $\mathbb{E}^n Y_t \to Y_t$
- Show that for n large enough a comparison theorem holds.
- Still there are issues...

$$\xi \to \mathbb{E}[\xi|\mathcal{F}^n]$$

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- f depends on parameters, conditional expectation does not necessarily define a measurable process. We need a version of $\mathbb{E}^n f$ progressively measurable in t and continuous in (y, z, u).

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- $\to$ taking optional projections $f^{n,o}$ with respect to $(\mathcal{P}^n)_n$ (progressive sets defined by X^n), is only possible for bounded or nonnegative processes.

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Then $\mathbb{E}F^K(v + X - X^n, t)\Big|_{v=X^n}$ defines a progressively measurable version of $(\mathbb{E}^n K(t))_t$

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