Book of abstracts

Université Savoie Mont Blanc

IAE, Annecy campus

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Contents

Monday

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Marco Fuhrman, 09:00 - 10:00 – Amphi 012

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Chairwoman : Monique Jeanblanc

A randomization method in optimal control and BSDEs with constrained jumps

In 2009 Bruno Bouchard introduced a control randomization procedure leading to an alternative representation of the value function of a Markovian optimal switching problem. In later years several authors further developed this approach with applications to many other stochastic optimization problems. We review this method and present various contexts where it applies. The main motivation is that it allows to represent the value of several optimization problems by means of a suitable BSDE, including cases when the associated Hamilton-Jacobi-Bellman equation is fully non-linear and even cases of non-Markovian control problems.

Dan Crisan, 10:00 - 10:30 – Amphi 012

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Chairwoman : Monique Jeanblanc

Infinite dimensional McKean-Vlasov processes. Application to fluid dynamics models

I will discuss a natural generalization of McKean-Vlasov (MKV) processes to infinite dimensional state spaces where the classical global Lipschitz condition may not hold in general. As an application, I will introduce an idealized Atmosphere-Ocean model that rests upon Hasselmann's paradigm for stochastic climate models. This is in-line with the famous quote by Robert Heinlein " Climate is what you expect, weather is what you get". In the model presented, the stochasticity is incorporated into the fast moving atmospheric component of an idealized coupled model by means of stochastic Lie transport, while the slow moving ocean model remains deterministic. In the MKV version of the model, the drift velocity of the stochastic vector field is replaced by its expected value. The talk is based on the paper [1].

[1] D. Crisan, D. D. Holm, P. Korn, An Implementation of Hasselmann's Paradigm for Stochastic Climate Modelling based on Stochastic Lie Transport, https://arxiv.org/abs/2205.04560v1

Parallel sessions, 11:00 - 12:30

Parallel sessions take place in Amphi 012, Amphi 108, Room 125 and Room 126. Sessions are:

- **FBSDEs in utility maximization games** – Amphi 012 (remote session)
  Chairman: Alexandre Popier
- **Connection with PDEs - 1** – Amphi 108
  Chairman: Serguey Nadtochiy
- **BSDEs, mean-field equations and irregularity** – Room 125
  Chairwoman: Christel Geiss
- **Numerical methods for BSDEs** – Room 126
  Chairman: Thomas Kruse

FBSDEs in utility maximization games

Organizers: Guanxing Fu , Alexandre Popier , Chao Zhou
Chairman : Alexandre Popier

Guanxing Fu
Department of Applied Mathematics,
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Mean field portfolio games with consumption

Using martingale optimality principle, we study mean field portfolio games with consumption. The Nash equilibrium is uniquely characterized by a mean field FBSDE, which is equivalent to a mean field BSDE. In particular, the optimal investment is characterized by the Z-component of the BSDE, and the optimal consumption is characterized by the Y-component of the BSDE. When market parameters are deterministic but time-dependent, Nash equilibrium in closed form is available .

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Relative wealth concerns with partial information and heterogeneous priors.

We establish a Nash equilibrium with N agents with the relative wealth performance criteria when the market return is unobservable. We show that the optimal investment strategy under a stochastic return rate model can be characterized by a fully-coupled FBSDE. We establish the existence and uniqueness result for the class of FBSDEs with stochastic coefficients and solve the utility game under partial information using deep neural networks. We demonstrate the efficiency and accuracy by a base-case comparison with the semi-analytical solution in the linear case. We examined the Sharpe ratios and the Variance Risk ratios (VRRs) by numerical simulation. We observe that the agent with the most accurate prior estimate is likely to lead the herd. Moreover, the effect of competition on heterogeneous agents varies more with market characteristics compared to the homogeneous case. Joint work with Chao Deng and Chao Zhou.

Chen Yang
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An Equilibrium Model for the Cross-Section of Liquidity Premia.

We study a risk-sharing economy where an arbitrary number of heterogenous agents trades an arbitrary number of risky assets subject to quadratic transaction costs. For linear state dynamics, the forward-backward stochastic differential equations characterizing equilibrium asset prices and trading strategies in this context reduce to a coupled system of matrix-valued Riccati equations. We prove the existence of a unique global solution and provide explicit asymptotic expansions that allow us to approximate the corresponding equilibrium for small transaction costs. These tractable approximation formulas make it feasible to calibrate the model to time series of prices and trading volume, and to study the cross-section of liquidity premia earned by assets with higher and lower trading costs. This is illustrated by an empirical case study. This is a joint work with Johannes Muhle-Karbe and Xiaofei Shi.

Connections with PDEs - 1

Contributed session
Chairman: Serguey Nadtochiy

Xiaolu Tan
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A C1 functional Itô formula and regularity of solution to PPDEs

Using Dupire's notion of vertical derivative, we provide a functional (path-dependent) extension of the Itô's formula of Gozzi and Russo (2006) that applies to C1-functions of continuous weak Dirichlet processes. It is motivated and illustrated by its applications to the hedging or superhedging problems of path-dependent options in mathematical finance. We then investigate the regularity of some path-dependent functionals, especially those from the solution of a class of path-dependent PDEs. This is based on joint works with Bruno Bouchard and Grégoire Loeper.

Jianjun Zhou
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Viscosity Solutions to Second Order Path-Dependent Hamilton-Jacobi-Bellman Equations and Applications

In this talk, a notion of viscosity solutions is introduced for second order path-dependent Hamilton-Jacobi-Bellman (PHJB) equations associated with optimal control problems for path-dependent stochastic differential equations. We identify the value functional of optimal control problems as unique viscosity solution to the associated PHJB equations. We also show that our notion of viscosity solutions is consistent with the corresponding notion of classical solutions, and satisfies a stability property. Applications to backward stochastic Hamilton-Jacobi-Bellman equations are also given.

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Well-posedness of path-dependent semilinear parabolic master equations

Master equations are partial differential equations for measure-dependent unknowns, and are introduced to describe asymptotic equilibrium of large scale mean-field interacting systems, especially in games and control theory. In this paper we introduce new semilinear master equations whose unknowns are functionals of both paths and path measures. They include state-dependent master equations, path-dependent partial differential equations (PPDEs), history information dependent master equations and time inconsistent (e.g. time-delayed) equations, which naturally arise in stochastic control theory and games. We give a classical solution to the master equation by introducing a new notation called strong vertical derivative (SVD) for path-dependent functionals, inspired by Dupire's vertical derivative, and applying stochastic forward-backward system argument. Moreover, we consider a class of non-smooth cases via a functional mollifying method.

BSDEs, mean-field equations and irregularity

Organizers: Christel Geiss
Chairwoman : Christel Geiss

Stefan Geiss
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Functional fractional smoothness of SDEs

We study functional fractional smoothness properties of stochastic differential equations. Our techniques are based on the decoupling method introduced in [1]. The approach in this talk uses potentials to describe the regularity of the coefficients and allows for a unified treatment, besides of standard SDEs, of McKean-Vlasov SDEs and associated Kusuoka-Stroock processes, stopped diffusions, local times, and more. As a special case we obtain embedding theorems for real interpolation spaces on the Wiener space for McKean-Vlasov SDEs.

This is joint work with Christel Geiss (University of Jyvaskyla).

[1] Stefan Geiss and Juha Ylinen, *Decoupling on the Wiener Space, Related Besov Spaces, and Applications to BSDEs.* Memoirs AMS 1335 (2021)

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L^p-Solutions to BSDEs with Jumps: Existence, Uniqueness and Comparison

We study the existence and uniqueness of solutions to backward stochastic differential equations driven by a Brownian motion and a Poisson random measure. In the L^p-setting, $p \geq 2$, we find a positive answer, assuming generators that satisfy an extension of the usual monotonicity condition (Osgood condition) including generators of the continuity type as $y \mapsto -|y| \log(|y|)$ possesses at 0. The growth in the y-variable is not restricted to linearity. In the delicate case $1 < p < 2$, we show unique solvability in the usual L^p spaces for solutions as well, however, the same assumptions on the generator demand new and different proof techniques. In both cases, in dimension 1, if the behavior of the generator's jump variable meets an additional, natural condition, then a comparison theorem holds.

This joint work with Stefan Kremsner (University of Graz).

Dan Crisan
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Modelling multi-period carbon markets using singular forward backward SDEs

I will introduce a model about the evolution of emissions and the price of emissions allowances in a carbon market such as the European Union Emissions Trading System (EU ETSP). The model accounts for multiple trading periods (and phases) and multiple times at which compliance can occur. At the end of each trading period, the participating firms must surrender allowances for the emissions made during that period, but any excess allowances can be used for compliance in the following periods. We show that the multi-period allowance pricing problem is well-posed for various mechanisms linking the trading periods (such as banking, borrowing and withdrawals). The results are based on the analysis of a forward-backward stochastic differential equation with the following special characteristics:
i. the forward and backward components are coupled,
ii. the final condition is singular and
iii. the forward component of the model is degenerate.

I will also introduce an infinite period model, that is, a model for carbon market with a sequence of compliance times and no end date. I will show that, under appropriate conditions, the value function for the multi-period pricing problem converges, as the number of periods increases, to a value function for this infinite period model.

This is joint work with Jean-Francois Chassagneux (Paris Diderot) and Hinesh Chotai (Citybank).

Numerical Methods for BSDEs

Organizer : Thomas Kruse
Chairman : Thomas Kruse

Stefan Ankirchner
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Solving fully coupled FBSDEs by minimizing a directly calculable error functional

We present a new scheme for approximating solutions of forward-backward stochastic differential equations (FBSDEs). The scheme is particularly suitable for fully coupled FBSDEs. The approximation relies on a piecewise in time approximation by minimizing an error functional that measures how well a process triplet satifies the FBSDE. The error functional is minimized in a finite-dimensional linear space based on iterated integrals. We provide sufficient conditions for the approximations to converge at the rate $N^{-1/2}$, where N is the time discretization parameter.

The talk is based on a joint work with Alexander Fromm (Jena).

Christian Bender
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“Regression anytime” with brute-force SVD truncation

We propose a new least-squares Monte Carlo algorithm for the approximation of conditional expectations in the presence of stochastic derivative weights. The algorithm can serve as a building block for solving dynamic programming equations, which arise, for example, in nonlinear option pricing problems or in time-discretization schemes for backward stochastic differential equations. Our algorithm can be generically applied when the underlying dynamics stem from an Euler approximation to a stochastic differential equation. A built-in variance reduction ensures that the convergence in the number of samples to the true regression function takes place at an arbitrarily fast polynomial rate, if the problem under consideration is smooth enough.

The talk is based on joint work with Nikolaus Schweizer (Tilburg).

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Gradient boosting-based numerical methods for high-dimensional backward stochastic differential equations

In this work we propose new algorithms for solving high-dimensional backward stochastic differential equations (BSDEs). Based on the general theta-discretization for the time-integrands, we show how to efficiently use eXtreme Gradient Boosting (XGBoost) regression to approximate the resulting conditional expectations in a quite high dimension. A rigorous analysis of the convergence is provided. Numerical results illustrate the efficiency and accuracy of our proposed algorithms for solving very high-dimensional (up to 10000 dimensions) nonlinear BSDEs. Notably, our new algorithms works also quite well on the problems with highly complex structure in high dimension, which cannot be tackled with the state-of-art numerical methods.

Parallel sessions, 14:00 - 16:00

Parallel sessions take place in Amphi 012, Amphi 108, Room 125 and Room 126. Sessions are:

- **Mean-Field Systems and applications - 1** – Amphi 012
  Chairman: Lukasz Szpruch
- **Connection with Economics and Finance** – Amphi 108
  Chairman: Peter Bank
- **Reflected BSDEs** – Room 125
  Chairwoman: Marie-Amélie Morlais
- **Numerical methods - 1** – Room 126
  Chairman: Christian Bender

Mean Field Systems and Applications 1

Contributed Session
Chairman: Lukasz Szpruch

William Hammersley
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Rearranged Stochastic Heat Equation

The aim of this work is to introduce noise to regularise gradient descents in the space of probability measures (motivated by mean-field learning algorithms). In contrast to the heuristic where noise is first introduced at the level of the mean field particle systems and one obtains a regularisation in the infinite particle regime, we begin by randomising directly the non-linear stochastic differential equation of McKean-Vlasov type. This is done by first fixing the underlying probability space as the unit circle with Lebesgue measure and considering the stochastic heat equation. In order to construct a noise that is more intrinsic to the original minimisation problem/gradient descent, we consider a discrete time scheme that, as one iteration, implements the stochastic heat equation with coloured noise over a small time interval and then composes with the symmetric non-increasing rearrangement. The resulting stochastic processes are constrained within a set functions in one-to-one correspondence with the quantile functions of a large class of random variables. As one refines the time discretisation it is expected that the dynamics should limit to a reflected SPDE, however in the absence of a corresponding integration by parts formula we are only able to identify the limit process' behaviour tested against a particular subclass of functions/processes. Nevertheless, this characterisation is well-posed and we demonstrate the regularising effects of our limiting dynamics. This is joint work with François Delarue.

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Bernt Øksendal
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Mean-field backward stochastic differential equations and applications

In this paper we study the linear mean-field backward stochastic differential equations (mean-field BSDE) of the form

$$dY(t) = -[\alpha_1(t)Y(t) + \beta_1(t)Z(t) + \int_{\mathbb{R}_0}\int_{\mathbb{R}_0} \eta_1(t,\zeta)K(t,\zeta)\nu(d\zeta) + \alpha_2(t)\mathbb{E}[Y(t)]$$

$$+\beta_2(t)\mathbb{E}[Z(t)]+\int_{\mathbb{R}_0}\int_{\mathbb{R}_0} \eta_2(t,\zeta)\mathbb{E}[K(t,\zeta)]\nu(d\zeta)+\gamma(t)]dt+Z(t)dB(t)+\int_{\mathbb{R}_0}\int_{\mathbb{R}_0} K(t,\zeta)\tilde{N}(dt,d\zeta), t\in[0,T],$$

$$Y(T) = \xi.$$

where (Y, Z, K) is the unknown solution triplet, B is a Brownian motion, $\tilde{N}$ is a compensated Poisson random measure, independent of B.

We prove the existence and uniqueness of the solution triplet (Y, Z, K) of such systems. Then we give an explicit formula for

the first component $Y(t)$ by using partial Malliavin derivatives.

To illustrate our result we apply them to study a mean-field recursive utility optimization problem in finance.

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Mean field stochastic differential equations with a discontinuous diffusion coefficient

We consider R^d-valued mean field stochastic differential equations of the type

$$X_t = x_0 + \int_0^t \sigma(s, X_s, \|X_s\|_{L_p}) dB_s + \int_0^t b(s, X_s, \|X_s\|_{L_p}) ds$$

with infinite time horizon, where $B = (B_t)_{t \in [0,\infty)}$ is a d-dimensional Brownian motion. We discuss the existence of a solution in the case that the diffusion and drift coefficient satisfy standard assumptions, but with the main exception that the diffusion coefficient σ is discontinuous in the L_p component.

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Submodular mean field games: Existence and approximation of solutions

We study mean field games with scalar Itô-type dynamics and costs that are submodular with respect to a suitable order relation on the state and measure space. The submodularity assumption has a number of interesting consequences. Firstly, it allows us to prove existence of solutions via an application of Tarski's fixed point theorem, covering cases with discontinuous dependence on the measure variable. Secondly, it ensures that the set of solutions enjoys a lattice structure: in particular, there exist a minimal and a maximal solution. Thirdly, it guarantees that those two solutions can be obtained through a simple learning procedure based on the iterations of the best-response-map. The mean field game is first defined over ordinary stochastic controls, then extended to relaxed controls. Our approach allows also to treat submodular mean field games with common noise.

Connections with economics and finance

contributed session
Chairman: Peter Bank

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A Mean Field Game Approach to Optimal Execution with Asset Bubble

Asset bubbles have attracted increasing attention, thanks to the emergence of meme stocks and cryptocurrency. We propose a mean field game (MFG) model on the classical problem of optimal execution but in the presence of an asset bubble. Traders become aware of the bubble potentially at different times and enter the game with the intention to take advantage of the positive drift. The obvious risk of "riding a bubble" is that the bubble will crash at a random time when the price of the asset suddenly plummets.

We consider two types of bubble bursts: an endogenous burst which results from excessive selling, and an exogenous burst which is independent from the actions of the traders. We adopt the method of progressive enlargement of filtrations, commonly used to analyze credit risks, which models the exogenous burst as a totally inaccessible stopping time. We define MFG equilibrium with random entry time in terms of probability kernels, and prove existence of MFG equilibrium using the weak formulation of MFGs.

This talk is based on a joint work with Ludovic Tangpi.

Xun Li
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Optimal Consumption with Loss Aversion and Reference to Past Spending Maximum

This talk studies an optimal consumption problem for a loss-averse agent with reference to past consumption maximum. To account for loss aversion on relative consumption, an S-shaped utility is adopted that measures the difference between the non-negative consumption rate and a fraction of the historical spending peak. We consider the concave envelope of the realization utility with respect to consumption, allowing us to focus on an auxiliary HJB variational inequality on the strength of concavification principle and dynamic programming arguments. By applying the dual transform and smooth-fit conditions, the auxiliary HJB variational inequality is solved in closed-form piecewisely and some thresholds of the wealth variable are obtained. The optimal consumption and investment control of the original problem can be derived analytically in the piecewise feedback form.

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Robust maximization of recursive utility with penalization on the model.

We show that robust control under volatility uncertainty in a continuous time setup allows for a dynamic description in terms of second order backward differential equations (2BSDE). The relevant penalizing mechanism is modeled by the distance of any two stochastic volatility models. Apart from unique existence of related utility processes, we establish the related Bellman–Equation.

[1] Name, Surname and Name2, Surname2 *Title of the work.* Journal, Year

[2] Name, Surname and Name2, Surname2 *Title of the work.* Journal, Year

Mehdi Talbi
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Sannikov's Principal-Agent problem with multiple agents

We study an extension of the well-known Sannikov's Principal-Agent problem to the multi-agents case. After briefly studiying the problem with N agents, we formulate the mean-field limit: given a contract offered by the principal, the competitive agents face a mean-field game, whose Nash equlibrium is anticipated by the principal. As the contract is composed of a payment and a retirement time, the principal faces a mean-field control-and-stopping problem, that we are able to characterize by an obstacle problem on Wasserstein space. Under some assumptions on the cost function of the agents, we show that the principal's problem reduces to a simple mean-field control problem, for which we propose some numerical results.

Reflected BSDEs

Contributed session
Chairwoman: Marie-Amélie Morlais

Adrien Richou
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Reflected BSDEs in non-convex domains

I will present some new results about the well-posedness of reflected backward differential equations in some non-convex domains that satisfy a so-called "weak star-shaped property". In particular we are able to establish some existence and uniqueness results in a Markovian framework with Hölder-continuous generator and terminal condition, but also in a general setting under a smallness assumption on the input data. Some links with martingale on manifolds will be presented at the end.

This is a joint work with Jean-François Chassagneux (Université de Paris) and Sergey Nadtochiy (Illinois Institute of Technology)

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Stochastic optimal switching and systems of variational inequalities with interconnected obstacles

This paper studies a system of m variational inequalities with interconnected obstacles in infinite horizon associated to optimal multi-modes switching problems. Our main result is the existence and uniqueness of a continuous solution in viscosity sense, for that system. The proof of the main result strongly relies on the connection between the systems of variational inequalities and reflected backward stochastic differential equations (RBSDEs) with oblique reflection, which will be characterized through a Feynman-Kac's formula. The main feature of our system of infinite horizon RBSDEs is that its components are interconnected through both the generators and the obstacles.

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Reflected and Doubly RBSDEs with Irregular Obstacles and a Large Set of Stopping Strategies

We introduce a new formulation of reflected BSDEs and doubly reflected BSDEs associated with irregular obstacles. In the first part of the paper, we consider an extension of the classical optimal stopping problem over a larger set of stopping systems than the set of stopping times (namely, the set of split stopping times), where the payoff process ξ is irregular and in the case of a general filtration. We show that the value family can be aggregated by an optional process v, which is characterized as the Snell envelope of the reward process ξ over split stopping times. Using this, we prove the existence and uniqueness of a solution Y to irregular reflected BSDEs. In the second part of the paper, motivated by the classical Dynkin game with completely irregular rewards considered by Grigorova et al. (2018), we generalize the previous equations to the case of two reflecting barrier processes. Under a general type of Mokobodzki's condition, we show the existence of the solution through a Picard iteration method and a Banach fixed point theorem.

Numerical methods - 1

Contributed Session
Chairman: Christian Bender

Emmanuel Gobet
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Newton method for stochastic control problems

We develop a new iterative method based on Pontryagin principle to solve stochastic control problems. This method is nothing else than the Newton method extended to the framework of stochastic optimal control, where the state dynamics is given by an ODE with stochastic coefficients and the cost is random. Each iteration of the method is made of two ingredients: computing the Newton direction, and finding an adapted step length. The Newton direction is obtained by solving an affine-linear Forward-Backward Stochastic Differential Equation (FBSDE) with random coefficients. This is done in the setting of a general filtration. Solving such an FBSDE reduces to solving a Riccati Backward Stochastic Differential Equation (BSDE) and an affine-linear BSDE, as expected in the framework of linear FBSDEs or Linear-Quadratic stochastic control problems. We then establish convergence results for this Newton method. In particular, Lipschitz-continuity of the second-order derivative of the cost functional is established with an appropriate choice of norm and under boundedness assumptions, which is sufficient to prove (local) quadratic convergence of the method in the space of uniformly bounded processes. To choose an appropriate step length while fitting our choice of space of processes, an adapted backtracking line search method is developed. We then prove global convergence of the Newton method with the proposed line search procedure, which occurs at a quadratic rate after finitely many iterations. An implementation with regression techniques to solve BSDEs arising in the computation of the Newton step is developed. We apply it to the control problem of a large number of batteries providing ancillary services to an electricity network.

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Convergence Rates of Random Walk Approximations of Forward-Backward SDEs

For the forward-backward SDE

$$X_t = x + \int_0^t b(r, X_r)dr + \int_0^t d(r, X_r)dB_r$$

$$Y_t = g(X_T) + \int_t^T f(s, X_s, Y_s, Z_s)ds + \int_t^T Z_s dB_s,\ 0 \le t \le T$$

Briand, Delyon and Memin have shown in 2001 a Donsker-type theorem: If one approximates the Brownian motion B by a random walk B^n, the according solutions (X^n, Y^n, Z^n) converge weakly to (X, Y, Z). [1]

If the random walk is constructed from the underlying Brownian motion via Skorohod embedding it can be shown that (X_t^n, Y_t^n, Z_t^n) converges to (X_t, Y_t, Z_t) in L_2. We estimate the rate of convergence in dependence of smoothness properties of the coefficients b and σ, the terminal condition function g and the generator f. However, the error between the random walk obtained by Skorohod embedding and the Brownian motion in L_2 is of the order $n^{-1/4}$. This affects the convergence of the forward-backward SDE. [2]

The approximation of the Brownian motion by a random walk in Wasserstein distance is of order $n^{-1/2}$, which implies also better rates in Wasserstein distance for the according forward-backward SDEs. [3]

The talk is based on joint results with Philippe Briand, Stefan Geiss, Céline Labart and Antti Luoto.

[1]. P. Briand, B. Delyon, and J. Mémin. Donsker-Type theorem for BSDEs. Electron. Commun. Probab., 6(1):1–14 (2001).

[2]. C. Geiss, C. Labart, and A. Luoto, Mean square rate of convergence for random walk approximation of forward-backward SDEs. Adv. in Appl. Probab. 52(3): 735 – 771 (2020).

[3]. P. Briand, C. Geiss, S. Geiss and C. Labart. Donsker-Type Theorem for BSDEs: Rate of Convergence. Bernoulli, 27(2): 899–929 (2021)

Jean-François Chassagneux
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Numerical approximation of singular FBSDEs: application to carbon market

In this talk, I will present a mathematical modeling of carbon markets based as introduced in Carmona, Delarue, Espinosa and Touzi (2013). These are cap and trade schemes, where a regulator sets a cap on the total amount of carbon emissions for all the market participants. The models lead to the studies of singular Forward-Backward SDEs, see e.g. Carmona and Delarue (2013). The theoretical and numerical study of this equation is difficult due to the degeneracy of the forward component and the discontinuity of the terminal condition. Observing the failure of "classical" FBSDEs schemes, we design new algorithms based on a splitting method to approximate the solution of such equation. I will present some theoretical results for the schemes convergence and numerical experiments which validate their practical efficiency.

This talk is based on joint works with Mohan Yang (Université de Paris).

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Wanqing Wang
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Convergence rate for pricing of American Option by penalisation method and its Monte-Carlo approximation

In this paper, we study the pricing of American Option taking advantage of the link with Reflected Backward Stochastic Differential equation (RBSDE in short). If we set the payoff as the reflecting obstacle S, a RBSDE generally writes as

$$Y_t = S_T + \int_t^T f(s, Y_s, Z_s)\mathbf{d}s + K_T - K_t - \int_t^T Z_s \mathbf{d}B_s, \quad 0 \le t \le T,$$
$$Y_t \ge S_t, \quad 0 \le t \le T,$$
$$\int_0^T (Y_t - S_t)\mathbf{d}K_t = 0,$$

where the driver f stands for the self-financing properties and the possible market fictions.

At time t, the price of American option $ess \sup_{\tau \in \mathcal{T}_{t,T}} \mathbb{E}\left[e^{-\int_t^\tau r_s \mathbf{d}s} S_\tau | \mathcal{F}_t\right]$ coincides with Y_t for some specific driver f when we deal with Itô market.

The solution to (1) can be approximated by a penalised BSDE (PBSDE in short) under the form

$$Y_t^\lambda = S_T + \int_t^T f(s, Y_s^\lambda, Z_s^\lambda)\mathbf{d}s + \lambda \int_t^T (Y_s^\lambda - S_s)^- \mathbf{d}s - \int_t^T Z_s^\lambda \mathbf{d}B_s.$$

Our main contribution is to prove that the convergence of $(Y^\lambda, Z^\lambda, K^\lambda)$ to (Y, Z, K) holds at rate $\frac{1}{\lambda}$ or $\frac{1}{\sqrt{\lambda}}$ as $\lambda \to +\infty$ according to some appropriate convex properties of S.

In addition, we design a discrete-time numerical scheme for PBSDE and prove its rate of convergence in term of the time-step h and λ. Results are illustrated using Monte Carlo simulations.

Parallel sessions, 16:30 - 18:00

Parallel sessions take place in Amphi 012, Amphi 108, Room 125 and Room 126. Sessions are:

- **BSDEs, Enlargement of Filtrations and Information in Finance** – Amphi 012
  Chairmen: Paolo Di Tella and Monique Jeanblanc
- **BSDEs in trade execution problems and related topics** – Amphi 108
  Chairman: Mikhail Urusov
- **From control theory to Deep Learning and back - 1** – Room 125
  Chairmen: Lukasz Szpruch and David Siska
- **Mean-field systems** – Room 126
  Chairman: Etienne Tanré

BSDEs, Enlargement of Filtrations and Information in Finance

Organizers: Paolo Di Tella , Monique Jeanblanc
Chairmen: Paolo Di Tella and Monique Jeanblanc

Claudia Ceci
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Optimal reinsurance via BSDEs in a partially observable contagion model

We study an optimal reinsurance problem under the criterion of maximizing the expected utility of terminal wealth when the loss process exhibits jump clustering features and the insurance company has restricted information about the claims arrival intensity. By solving the associated filtering problem we reduce the original problem to a stochastic control problem under full information. Since the classical Hamilton-Jacobi-Bellman approach does not apply, due to the infinite dimensionality of the filter, we choose an alternative approach based on Backward Stochastic Differential Equations (BSDEs). Precisely, we characterize the value process and the optimal reinsurance strategy in terms of a BSDE driven by a marked point process. The talk is based on a joint work with M. Brachetta, G. Callegaro and C. Sgarrra.

Elena Bandini
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Progressively Enlargement of Filtrations and Control Problems for Step Processes

In the present talk we address stochastic optimal control problems for a step process $(X, \mathbb{F})$ under a progressive enlargement of the filtration. The global information is obtained adding to the reference filtration $\mathbb{F}$ the point process $H = 1_{[\tau,+\infty)}$. Here τ is a random time that can be regarded as the occurrence time of an external shock event. We study two classes of control problems, over $[0, T]$ and over the random horizon $[0, T \wedge \tau]$. We solve these control problems following a dynamical approach based on a class of BSDEs driven by the jump measure μ^Z of the semimartingale $Z = (X, H)$, which is a step process with respect to the enlarged filtration $\mathbb{G}$. The BSDEs that we consider can be solved in $\mathbb{G}$ thanks to a martingale representation theorem which we also establish here. To solve the BSDEs and the control problems we ensure that Z is quasi-left continuous in the enlarged filtration $\mathbb{G}$. Therefore, in addition to the $\mathbb{F}$-quasi left continuity of X, we assume some further conditions on τ. This is a joint work with Fulvia Confortola (Politecnico di Milano) and Paolo Di Tella (Technische Universitaet Dresden).

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Risk measures and progressive enlargement of filtrations: a BSDE approach

From the beginning of the 21st century, connections between dynamic risk measures and Backward Stochastic Differential Equations (or BSDEs, for short) have been studied in the literature. The theory of g-expectations, developed by S. Peng (see, e.g., [4]), paved the way for this connection, that has been thoroughly studied when the noise driving BSDEs is either a Brownian motion (see, e.g., [1, 6]) or a Brownian motion and an independent Poisson random measure (as in [5]).

Here we consider a class of BSDEs with jumps (BSDEJ) introduced by I. Kharroubi and T. Lim in [3], whose driving noise is given by a Brownian motion and a (not necessarily independent) marked point process. The existence and uniqueness results provided in [3], allow us to define dynamic risk measures induced by such BSDEJs.

From a financial perspective, these BSDEJs and, consequently, the induced dynamic risk measures, can be adopted to evaluate the riskiness of a financial position whose future value may be influenced by sudden random events, modeled by the marked point process driving the BSDEJ. Another important feature is that the information available to financial agents is progressively updated as these random events occur. This feature is mathematically encoded in the progressive enlargement of a Brownian *reference filtration*. It is proved in [3] that under such a framework it is possible to provide a decomposition of the solution to the BSDEJs into processes that are solution, between each pair of consecutive random times, of BSDEs driven only by the Brownian motion.

We show that a similar decomposition holds also for the induced dynamic risk measure. Furthermore, we prove that properties of the driver of the BSDEJ are reflected into desirable properties of the induced risk measure, such as monotonicity, convexity, and time-consistency, to name a few. Finally, we focus on its dual representation and provide some examples.

This is joint work with Emanuela Rosazza Gianin.

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[3] I. Kharroubi and T. Lim. Progressive enlargement of filtrations and backward stochastic differential equations with jumps. *J. Theoret. Probab.* **27(3)**, 683–724, 2014.

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[5] M. C. Quenez and A. Sulem. BSDEs with jumps, optimization and applications to dynamic risk measures. *Stochastic Process. Appl.* **123(8)**, 3328–3357, 2013.

[6] E. Rosazza Gianin. Risk measures via g-expectations. *Insurance Math. Econom.* **39(1)**, 19–34, 2006.

BSDEs in trade execution problems and related topics

Organizer : Mikhail Urusov
Chairman : Mikhail Urusov

Julia Ackermann
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Optimal trade execution in a stochastic order book model

To analyze an optimal trade execution problem, we set up a limit order book model in continuous time of Obizhaeva-Wang type. In order to model stochastic liquidity, we allow both order book depth and resilience to evolve randomly in time. Execution strategies may have infinite variation. We find that, under appropriate assumptions, minimal execution costs and optimal execution strategies are characterized by a quadratic BSDE.

This talk is based on joint work with Thomas Kruse and Mikhail Urusov.

Peter Bank
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What if we (sort of) new what the future brings?

We study optimal investment problems for an insider who receives noisy information about short-term stock price movements, but cannot arbitrarily take advantage of this knowledge because of quadratic transaction costs. In the Bachelier setting with exponential utility, we give an explicit solution to this control problem with intrinsically infinite-dimensional state variable by solving its dual problem.

The talk is based on joint work with Yan Dolinsky and Miklos Rasonyi.

Ulrich Horst
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Optimal trade execution under small market impact and portfolio liquidation with semimartingale strategies

We consider an optimal liquidation problem with instantaneous price impact and stochastic resilience for small instantaneous impact factors. Within our modelling framework, the optimal portfolio process converges to the solution of an optimal liquidation problem with general semimartingale controls when the instantaneous impact factor converges to zero. Our results provide a unified framework within which to embed the two most commonly used modelling frameworks in the liquidation literature and provide a microscopic foundation for the use of semimartingale liquidation strategies and the use of portfolio processes of unbounded variation. Our convergence results are based on novel convergence results for BSDEs with singular terminal conditions and novel representation results of BSDEs in terms of uniformly continuous functions of forward processes.

The talk is based on joint work with Evgueni Kivman.

From control theory to Deep Learning and back 1

Organizers : David Šiška , Lukasz Szpruch
Chairman : David Šiška

Xin Guo
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GANs training, a stochastic control approach

There is a rapidly increasing interest in applying GANs for financial data generation. GANs training, however, is known to suffer various issues including vanishing gradients, model collapses, and difficulty in convergence. In this talk, we discuss a new stochastic control approach for GANs training. We will analyze the associated minimax games via weak dynamic programming principle and derive explicit solutions for the optimal learning rate and the batch size choices. We will show through numerical experiment the advantage of our control approach.

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Deep Learning for Principal Agent Mean Field Games

We develop a deep learning algorithm for solving Principal-Agent (PA) mean field games with market-clearing conditions – a class of problems that have thus far not been studied and one that poses difficulties for standard numerical methods. We use an actor-critic approach to optimization, where the agents form a Nash equilibria according to the principal's penalty function, and the principal evaluates the resulting equilibria. The inner problem's Nash equilibria is obtained using a variant of the deep backward stochastic differential equation (BSDE) method modified for McKean-Vlasov forward-backward SDEs that includes dependence on the distribution over both the forward and backward processes. The outer problem's loss is further approximated by a neural net by sampling over the space of penalty functions. We apply our approach to a stylized PA problem arising in Renewable Energy Certificate (REC) markets, where agents may rent clean energy production capacity, trade RECs, and expand their long-term capacity to navigate the market at maximum profit. Our numerical results illustrate the efficacy of the algorithm and lead to interesting insights into the nature of optimal PA interactions in the mean-field limit of these markets. Further, we prove the convergence of our approach to the true solution.

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Convergence of policy gradient for entropy regularized MDP with neural network approximation in the mean-field regime

We study the global convergence of policy gradient for infinite-horizon, *continuous state and action space*, entropy-regularized Markov decision processes (MDPs). Here, we consider a softmax policy with (one-hidden layer) neural network approximation in a mean-field regime. We add further entropic regularization in terms of the associated mean-field probability measure and study the corresponding gradient flow in the 2-Wasserstein metric. We show that if the regularization in terms of the mean-field measure is sufficient, then the gradient flow converges exponentially fast to the unique stationary solution, which is the unique maximizer of the regularized MDP objective. Our non-asymptotic analysis shows how entropic regularization affects the search for the optimal policy. This extends the pioneering work of [1] and [2], where the authors establish non-asymptotic convergence of policy gradient for entropy-regularized MDPs in the tabular setting.

[1] Song Mei and Andrea Montanari and Phan-Minh Nguyen *A mean field view of the landscape of two-layer neural networks.* PNAS, 2018

[2] Alekh Agarwal and Sham M Kakade and Jason D Lee and Gaurav Mahajan *Optimality and approximation with policy gradient methods in Markov decision processes.* Conference on Learning Theory, 2020

Mean Field Systems

Organizer : Etienne Tanré
Chairman : Etienne Tanré

François Delarue
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Uniform in time weak propagation of chaos on the torus. Application to Kuramoto's model.

We address the long time behaviour of weakly interacting diffusive particle systems on the d-dimensional torus. Our main result is to show that, under certain regularity conditions, the weak error between the empirical distribution of the particle system and the theoretical law of the limiting process (governed by a McKean-Vlasov stochastic differential equation) is of the order $O(1/N)$, uniform in time on $[0, \infty)$, where N is the number of particles in the interacting diffusion. This comprises general interaction terms with a small enough mean-field dependence together with interactions terms driven by an H-stable potential. Our approach relies on a systematic analysis of the long-time behaviour of the derivatives of the semigroup generated by the McKean-Vlasov SDE, which may be explicitly computed through the linearised Fokker-Planck equation. Ergodic estimates for the latter hence play a key role in our analysis. We believe that this strategy is flexible enough to cover a wider broad of situations. To wit, we succeed in adapting it to the super-critical Kuramoto model, for which the corresponding McKean-Vlasov equation has several invariant measures.

Joint work with Alvin Tse.

Wilhelm Stannat
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Mean-field approach to Bayesian estimation of Markovian signals

Estimating Markovian signals X from noisy observations is an important problem in the natural and engineering sciences. Within the Bayesian approach the underlying mathematical problem essentially consists in the (stochastic) analysis of the conditional law of X with a view towards its efficient numerical approximation.

In this talk I will discuss mean-field type descriptions of the conditional law of X, when X is the solution of a stochastic differential equation, and present recent results on corresponding ensemble-based numerical approximations.

The talk is based on joint work with T. Lange, S. Pathiraja and S. Reich.

1. S. Pathiraja, S. Reich, W. Stannat: McKean-Vlasov SDEs in nonlinear filtering, SIAM J. Control Optim. 59 (2021), no. 6, 4188–4215.
2. S. Pathiraja, W. Stannat: Analysis of the feedback particle filter with diffusion map based approximation of the gain, Foundations of Data Science 3 (2021): 615-645.

3. T. Lange, W. Stannat: Mean field limit of Ensemble Square Root Filters - discrete and continuous time, Foundations of Data Science 3 (2021): 563-588.

Milica Tomasevic
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Quantitative particle approximation of nonlinear Fokker-Planck equations with singular kernel

In this talk, we are interested in the convergence of the empirical measure of moderately interacting particle systems with singular interaction kernels. First, we obtain the quantitative convergence of the time marginals of the empirical measure of particle positions towards the solution of the limiting nonlinear Fokker-Planck equation. Second, we show the well-posedness for the McKean-Vlasov SDE involving such singular kernels and the convergence of the empirical measure towards it (propagation of chaos).

These results only require very weak regularity on the interaction kernel, including the Biot-Savart kernel, and attractive kernels such as Riesz and Keller-Segel kernels in arbitrary dimension. For some of these examples, this is the first time that a quantitative approximation of the PDE is obtained by means of a stochastic particle system, which paves the way to numerical applications. In particular, this convergence still holds (locally in time) for PDEs exhibiting a blow-up in finite time.

The techniques used in [1] to prove the above results are based on a semigroup approach combined with a fine analysis of the regularity of infinite-dimensional stochastic convolution integrals.

[1] C. Olivera, A. Richard and M. Tomasevic *Quantitative particle approximation of nonlinear Fokker-Planck equations with singular kernel.* Accepted for publication in Annals of Scuola Normale di Pisa, 2021+

Tuesday

Contents

Shige Peng, 09:00 - 10:00

Scool of Mathematics, Shandong University, China
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Chairman : Etienne Pardoux

Some problems in SDEs, BSDEs and their corresponding PDEs

In this talk we discuss some properties of stochastic differential equations(SDEs), backward SDEs and the related partial differential equations.

Parallel sessions, 10:30 - 12:00

Parallel sessions take place in Amphi 012, Amphi 108, Room 125 and Room 126. Sessions are:

- **Mean Field control problems and Related topics** – Amphi 012
  Chairman: Rainer Buckdahn
- **Backward stochastic Volterra integral equations** – Amphi 108
  Chairman: Jiongmin Yong
- **Numerical methods - 2** – Room 125
  Chairman: Jean-François Chassagneux
- **Connection with PDEs - 2** – Room 126
  Chairman: Huilin Zhang

Mean-field control problems and related topics

Organizers : Rainer Buckdahn , Juan Li
Chairman : Rainer Buckdahn

Dan Goreac
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State-constrained controlled mean-field flows

This talk focuses on controlled stochastic flows with mean-field dynamics complying with (closed) state restrictions. This property, known as (near)-viability, is tackled via (quasi-)tangency methods. Law restrictions and mixed state-law restrictions are considered as is the interplay between the two classes. Besides theoretical issues, specific applications to comparison in the convex order and/or normality illustrate the results.

Based on a joint work with Rainer Buckdahn (Université de Bretagne Occidentale, France) and Juan Li (Shandong University, China).

Juan Li
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A stochastic maximum principle for partially observed general mean-field control problems with only weak solution

In this talk we focus on a general type of mean-field stochastic control problems with partial observations, in which the coefficients of the systems depend in a nonlinear way not only on the paths but also on the conditional laws of the state with respect to the observation process up to date. We first deduce the well-posedness of the state-observation systems, with the help of methods of reference probability measure. Then we are devoted to investigating a Peng's stochastic maximum principle of the control problem, where the control space does not need to be convex. From the use of reference probability measure, we need to work with the law under different probability measures, which makes the computations very technical. It's worth mentioning that the first order variational equation we obtain is a new type of coupled mean-field SDE.

Based on a joint work with Chao Mi, Hao Liang (SDU, China).

Wenqiang Li
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On the value of a general stochastic differential game with ergodic payoff

In this talk a type of zero-sum stochastic differential games with ergodic payoff is considered, in which the diffusion system does not need to be non-degenerate. We first show the existence of a viscosity solution of the associated ergodic Hamilton-Jacobi-Bellman-Isaacs equation under the dissipativity condition. With the help of this viscosity solution, we then give the estimates for the upper and lower ergodic value functions by constructing a series of non-degenerate approximating processes and using the sup- and inf-convolution techniques. Finally, we obtain the existence of a value under the Isaacs condition and its representation formulae. In addition, we apply our results to study a type of pollution accumulation problems with the long-run average social welfare.

Based on a joint work with Juan Li (Shandong University, China) and Huaizhong Zhao (Durham University, UK).

Backward Stochastic Volterra Integral Equations

Organizers: Tianxiao Wang and Jiongmin Yong
Chairman : Jiongmin Yong

Yushi Hamaguchi
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Open-loop equilibrium controls in time-inconsistent stochastic recursive control problems

We investigate a time-inconsistent stochastic recursive control problem where the cost functional is defined by the solution to a backward stochastic Volterra integral equation (BSVIE, for short). We provide a necessary and sufficient condition for an open-loop equilibrium control via variational methods and show that the corresponding first- and second-order adjoint equations become the so-called extended BSVIEs.

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Backward stochastic Volterra integro-differential equations and applications in optimal control problem 2

Abstract: In this article, a class of backward stochastic Volterra integro-differential equations (BSVIDEs) is introduced and studied. It is worthy mentioning that the proposed BSVIDEs can not be covered by the existing backward stochastic Volterra integral equations (BSVIEs), and they also have the nice flow property such that Itô's formula becomes quite applicable. It is found that BSVIDEs can provide a neat sufficient condition for the solvability of BSVIEs with generator depending on the diagonal value of the solutions. As applications, the optimal control problems in terms of maximum principles and linear quadratic control problems of optimal control for forward stochastic Volterra integro-differential equations (FSVIDE) are investigated. In contrast with the BSVIEs in current literature, some interesting phenomena and advantages of BSVIDEs are revealed.

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On a Class of Reflected Backward Stochastic Volterra Integral Equations and Related Time-Inconsistent Optimal Stopping Problems 2

We introduce a class of one-dimensional continuous reflected backward stochastic Volterra integral equations driven by Brownian motion, where the reflection keeps the solution above a given stochastic process (lower obstacle). We prove existence and uniqueness by a fixed point argument and derive a comparison result. Moreover, we show how the solution of our problem is related to a time-inconsistent optimal stopping problem and derive an optimal strategy. This is joint work with Boualem Djehiche (KTH).

Numerical methods 2

Contributed Session
Chairman : Jean-François Chassagneux

Balint Negyesi
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The One Step Malliavin scheme: new discretization of BSDEs implemented with deep learning regressions

In this talk, I present results from my recent work (Negyesi et al., 2021) where a novel discretization is proposed for FBSDEs with differentiable coefficients, simultaneously solving the BSDE and its Malliavin sensitivity problem. It is known (El Karoui et al., 1997) that under suitable regularity assumptions the solution pair of a BSDE is differentiable in the Malliavin sense, and the Malliavin derivatives $\{(D_sY_t, D_sZ_t)\}_{0\leq s,t\leq T}$ satisfy a linear BSDE themselves, where the Z process admits to a continuous modification provided by $Z_t = D_tY_t$. The One Step Malliavin (OSM) exploits this relation and estimates the control process by the corresponding linear BSDE of the Malliavin derivatives, which implies the need to provide accurate Γ estimates. The approximation is based on a merged formulation given by the Feynman-Kac formulae and the Malliavin chain rule. The continuous time dynamics is discretized with a theta-scheme. A discrete time approximation error analysis is carried out to show L^2 convergence of order $1/2$, under standard Lipschitz assumptions and additive noise in the forward diffusion. The resulting discrete time approximations for the Z process have the same order of conditional variance as for the Y process, which makes the scheme more tractable in a regression Monte Carlo setting. However, the presence of Γ estimates induces additional difficulties. In order to allow for an efficient numerical solution of the arising semi-discrete conditional expectations in possibly high-dimensions, two fully-implementable schemes are considered: an extension to the BCOS method (Ruijter and Oosterlee, 2015) as a reference in the one-dimensional framework, and neural network Monte Carlo regressions in case of high-dimensional problems, similarly to the recently emerging class of Deep BSDE methods (Han et al., 2018; Huré et al., 2020). The total approximation error is investigated and shown to be consistent with the discretization under the assumption of perfectly converging gradient descent iterations. Numerical experiments are provided for a range of different semi-linear equations up to 50 dimensions, demonstrating that the proposed scheme yields a significant improvement in the control estimations.

Keywords: discrete time approximations, Malliavin calculus, Deep BSDE, gamma estimates, regression Monte Carlo, BCOS

References - Negyesi, B., Andersson, K., and Oosterlee, C. W. (2021). The One Step Malliavin scheme: new discretization of BSDEs implemented with deep learning regressions. arXiv:2110.05421. http://arxiv.org/abs/2110.05421 - El Karoui, N., Peng, S., and Quenez, M. C. (1997). Backward stochastic differential equations in finance. Mathematical finance, 7 (1), 1–71. https://doi.org/10.1111/1467-9965.00022 - Ruijter, M. J., and Oosterlee, C. W. (2015). A Fourier

Cosine Method for an Efficient Computation of Solutionsto BSDEs. SIAM Journal on Scientific Computing, 37 (2), A859–A889. https://doi.org/10.1137/130913183 - Han, J., Jentzen, A., and E, W. (2018). Solving high-dimensional partial differential equations using deep learning. Proceedings of the National Academy of Sciences, 115(34), 8505–8510. https://doi.org/10.1073/pnas.1718942115 - Huré, C., Pham, H., and Warin, X. (2020). Deep backward schemes for high-dimensional nonlinear PDEs. Mathematics of Computation, 89 (324), 1547–1579. https://doi.org/10.1090/mcom/3514

The corresponding article is a joint work with Kristoffer Andersson (Research Group of Scientific Computing, Centrum Wiskunde and Informatica) and Cornelis W. Oosterlee (Mathematical Institute, Utrecht University).

Lukasz Stepien and Michal Sobieraj
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On efficient approximation of solutions of SDEs driven by countably dimensional Wiener process and Poisson random measure

In this presentation we refer recent results [3] on optimal pointwise approximation of SDEs of the following form

$$\begin{cases} dX(t) = a(t, X(t))dt + b(t, X(t))dW(t) + \int\limits_{\mathcal{E}} c(t, X(t-), y)N(dy, dt), t \in [0, T], \\ X(0) = \eta, \end{cases}$$

where $T > 0$, $\mathcal{E} = \mathbf{R}^{d'} \setminus \{0\}$, $d' \in \mathbf{N}$, $W = [W_1, W_2, \ldots]^T$ is a countably dimensional Wiener process, and $N(dy, dt)$ is a Poisson random measure with an intensity measure $\nu(dy)dt$ (see [1], [2]). We assume that $\nu(dy)$ is a finite Lévy measure on $(\mathcal{E}, \mathcal{B}(\mathcal{E}))$.

In a certain class of coefficients $a : [0, T] \times \mathbf{R}^d \to \mathbf{R}^d$, $b : [0, T] \times \mathbf{R}^d \to l^2(\mathbf{R}^d)$, $c : [0, T] \times \mathbf{R}^d \to \mathbf{R}^d$ we investigate error of a truncated dimension randomized Euler scheme, which uses evaluations of finite number M of components of the Wiener process W. We establish upper bound on its error in the terms of the discretization parameter n and the truncation parameter M. In suitable subclasses we show also corresponding lower bounds on the error of an arbitrary algorithm that is based on (finite dimensional) evaluations of (a, b, c, W).

At the end we present results of numerical experiments performed on GPUs, where we used a suitable implementation (in CUDA C) of the truncated dimension randomized Euler algorithm.

[1] S. N. Cohen and R. J. Elliott. *Stochastic Calculus and Applications, 2nd. ed.* Springer, 2015.

[2] I. Gyöngy and N. V. Krylov. *On stochastic equations with respect to semimartingales I.* Stochastics, Volume 4, 1980,1–21.

[3] P. Przybylowicz and M. Sobieraj and L. Stepien. *Efficient approximation of SDEs driven by countably dimensional Wiener process and Poisson random measure,* accepted in SIAM Journal on Numerical Analysis (SINUM) in January, 2022.

Connections with PDEs - 2

Contributed session
Chairman : Huilin Zhang

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Probabilistic solutions to Stefan equations

This talk is concerned with the probabilistic methods for solving Stefan free-boundary PDEs (a.k.a. laplacian growth models). The latter equations appear in many models of fundamental physical and biological processes, such as: phase transition (i.e., melting/freezing), phase segregation (e.g., aging of alloys), crystal growth, neurons interaction, etc.. Despite their importance, to date, there exists no general existence and uniqueness theory for such equations due to the potential singularity of the solutions. Recently, the probabilistic methods, based on the analysis of associated mean-field particle systems and McKean-Vlasov equations, were successfully used to tackle the mathematical challenges that could not be addressed by the classical analytic methods, yielding new well-posedness results for certain types of Stefan equations. I will present an overview of the existing results and will focus on the existence and uniqueness problem of a solution to the Stefan equation with surface tension. This talk is based on joint works with F. Delarue, M. Shkolnikov, and X. Zhang.

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Coupled FBSDEs with Measurable Coefficients and its Application to Parabolic PDEs

Using purely probabilistic methods, we prove the existence and the uniqueness of solutions for a system of coupled forward-backward stochastic differential equations (FBSDEs) with measurable, possibly discontinuous coefficients. As a corollary, we obtain the well-posedness of semilinear parabolic partial differential equations (PDEs)

$$\mathcal{L}u(t,x) + F(t,x,u,\partial_x u) = 0; \qquad u(T,x) = h(x)$$
$$\mathcal{L} := \partial_t + \frac{1}{2}\sum_{i,j=1}^{m} (\sigma\sigma^\intercal)_{ij}(t,x)\partial^2_{x_i x_j}$$

in the natural domain of the second-order linear parabolic operator $\mathcal{L}$. We allow F and h to be discontinuous with respect to x. Finally, we apply the result to optimal policy-making for pandemics and pricing of carbon emission financial derivatives. This is a joint work with Yunxi Xu.

Elena Issoglio
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A Feynman-Kac result via Markov BSDEs with generalized drivers

In this talk I will investigate BSDEs where the driver contains a distributional term (in the sense of generalised functions) and derive general Feynman-Kac formulae related to these BSDEs. I will introduce an integral operator to give sense to the equation and then show the existence of a strong solution employing results on a related PDE. Due to the irregularity of the driver, the Y -component of a couple (Y, Z) solving the BSDE is not necessarily a semimartingale but a weak Dirichlet process. Based on a joint work with Francesco Russo (ENSTA).

Industrial Session, 13:30 - 15:30 – Amphi 012

Chairman : Emmanuel Gobet

David Šiška, 13:30 - 14:00

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System design validation and parameter optimisation through agent simulation

Vega protocol is a fully automated decentralised derivatives exchange allowing community-driven market creation.

Trading happens on classical limit order books with protective auctions.

The fully automated system features three interlocking parts: i) risk management based on stochastic models and expected shortfall for margin calculation and atomic liquidations, ii) liquidity provision mechanism incentivising "firm" liquidity irrespective of market conditions and iii) protective circuit breakers placing the market into auction when either large price moves happen or there isn't sufficient liquidity.

In this talk I will discuss how we used agent-based modelling with agents built on concepts from stochastic control / optimal market making and reinforcement learning to test, validate the above design and fine-tune various system parameters.

Clémence Alasseur, 14:00 - 14:30

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MFG model with a long-lived penalty at random jump times: application to demand side management for electricity contracts

We consider an energy system with n consumers who are linked by a Demand Side Management (DSM) contract, i.e. they agreed to diminish, at random times, their aggregated power consumption by a predefined volume during a predefined duration. Their failure to deliver the service is penalized via the difference between the sum of the n power consumptions and the contracted target. We are led to analyse a non-zero sum stochastic game with n players, where the interaction takes place through a cost which involves a delay induced by the duration included in the DSM contract. When n tends to infinity, we obtain a Mean-Field Game (MFG) with random jump time penalty and interaction on the control. We prove a stochastic maximum principle in this context, which allows to compare the MFG solution to the optimal strategy of a central planner. In a linear quadratic setting we obtain an semi-explicit solution through a system of decoupled forward-backward stochastic differential equations with jumps, involving a Riccati Backward SDE with jumps. We show that it provides an approximate

Nash equilibrium for the original n-player game for n large. Finally, we propose a numerical algorithm to compute the MFG equilibrium and present several numerical experiments.

Frédéric Abergel, 14:30 - 15:00

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Portfolio construction for multifactorial investment strategies

In this talk I will present a general framework for multifactorial investment strategies in asset management. The definitions and basic properties of some classical industry factors will be presented, and a recently developed risk model used for asset allocation will be discussed. The performance attribution will also be addressed. The talk is based on two papers, with references below.

[1] Performance attribution for multifactorial equity portfolios, Frédéric Abergel and Thomas Heckel, DOI: 10.21314/JOIS.2021.014

[2] A factor-based risk model for multifactor investment strategies, Frédéric Abergel, Benoit Bellone and François Soupé, DOI: 10.21314/JOR.2022.027

Julien Guyon, 15:00 - 15:30

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Volatility is (Mostly) Path-Dependent

We learn from data that volatility is mostly path-dependent: more than 90% of the variance of the implied volatility of equity indexes is explained endogenously by past index returns, and more than 70% for (noisy estimates of) future daily realized volatility. The path-dependency that we uncover is remarkably simple: a linear combination of a weighted sum of past daily returns and the square root of a weighted sum of past daily squared returns with different time-shifted power-law weights. This suggests a simple continuous-time path-dependent volatility model that may be fed historical or risk-neutral parameters. The weights can be approximated by superpositions of exponential kernels to produce Markovian models. In particular, we propose a five-dimensional Markovian PDV model which captures all the important stylized facts of volatility and produces remarkably realistic price and volatility paths.

Parallel sessions, 16:00 - 17:30

Parallel sessions take place in Amphi 012, Amphi 108, Room 125 and Room 126. Sessions are:

- **Quadratic BSDEs** – Amphi 012
  Chairman: Adrien Richou
- **Recent development for BSDEs and mean-field games in optimal control** – Amphi 108
  Chairman: Dylan Possamaï
- **Mean Field Systems with Applications in Economics and Finance** – Room 125
  Chairmen: Fulvia Confortola and Andrea Cosso
- **TBA** – Room 126
  Chairman: Miryana Grigorova

Quadratic BSDEs

Contributed Sessions
Chairman: Adrien Richou

Joe Jackson
The University of Texas at Austin, United States
jjackso1@utexas.edu

Quasilinear parabolic systems and FBSDEs with quadratic growth

In this talk I will present some recent results on Markovian FBSDE systems with quadratic growth. The focus is on the case that the unknown Y is multi-dimensional, the driver f has quadratic growth in z, and the volatility $\sigma = \sigma(t,x,y)$ depends on y. This is a qualitative jump from the case $\sigma = \sigma(t,x)$ (where results for quadratic FBSDEs have previously been obtained), since one can no longer formally reduce to a BSDE system via the Girsanov transform. The approach is to use new probabilistic techniques to obtain a-priori Hölder and then Lipschitz estimates for the corresponding quasilinear PDE system, and then use standard PDE arguments to infer existence.

Rhoss Beauneur Likibi Pellat
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Differentiability of quadratic forward-backward SDEs with non-Lipschitz drift

In this paper, we attempt to fill in the gap in the literature concerning both, the Classical and the variational differentiability in the sense of Malliavin calculus for a type of Markovian quadratic forward-backward SDEs (QFBSDEs for short) when the drift of the forward system does not satisfy the standard Lipschitz continuous assumption. The driver of the backward equation allows this type of nonlinearities: $f(|y|)|z|^2$, where f is any locally integrable function and the terminal value is a bounded functional of the forward process. In addition, we provide a convergence result of the numerical approximation of solutions to such QFBSDEs. We note that, the rate of convergence of our scheme is the same as in [**?**] i.e. when the drift is uniformly Lipschitz continuous and continuously differentiable with bounded derivatives in the spatial variable.

Daniel Bussell
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Some neural network schemes for the effective resolution of quadratic BSDES

We introduce an overparameterised feedforward neural network based scheme for the effective resolution of quadratic BSDEs with bounded terminal condition. We are inspired by earlier theoretical schemes that lacked practical implementation regarding the calculation of expectations. We rely on a mulit layer feedforward neural network with parameters defined by an Overparameterization Theorem. Some numerical examples are studied.

Recent development for BSDEs and mean-field games in optimal control

Organizers: Dylan Possamaï
Chairman: Dylan Possamaï

Thibaut Mastrolia
IEOR – University of California Berkeley
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Regulation of renewable resource exploitation

We investigate the impact of a regulation policy imposed on an agent exploiting a possibly renewable natural resource. We adopt a principal-agent model in which the Principal looks for a contract, i.e. taxes/compensations, leading the Agent to a certain level of exploitation. For a given contract, we first describe the Agent's optimal harvest using the BSDE theory. We then turn to the problem of the Principal by proposing a Krylov type approach to solve the problem inducing non linear PDE with irregular coefficients. In the second part of the talk, going from a scaling approach for birth/death processes, we investigate the convergence of values of stochastic control problems for discrete population models together with optimal controls.
Based on joint works with Idris Kharroubi (Sorbonne Unviersité), Thomas Lim (ENSIIE) and Paul Jusselin (École Polytechnique)

Julien Claisse
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Mean-field games with branching

Mean field games are concerned with the limit of large-population stochastic differential games where the agents interact through their empirical distribution. In the classical setting, the number of players is large but fixed throughout the game. However, in various applications, such as population dynamics or economic growth, the number of players can vary across time which may lead to different Nash equilibria. For this reason, we introduce a branching mechanism in the population of agents and obtain a variation on the mean field game problem. As a first step, we study a simple model using a PDE approach to illustrate the main differences with the classical setting. Then we study the problem in a general setting by a probabilistic approach, based upon the relaxed formulation of stochastic control problems. This is a joint work with Zhenjie Ren and Xiaolu Tan.

Dylan Possamaï
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BSVIEs, time-inconsistency, and contract theory

We address the problem of Moral Hazard in continuous time between a Principal and an Agent that has time-inconsistent preferences. Building upon previous results on non- Markovian time-inconsistent control for sophisticated agents, we are able to reduce the problem of the principal to a novel class of control problems, whose structure is intimately linked to the representation of the problem of the Agent via a so-called extended Backward Stochastic Volterra Integral equation. We will present some results on the characterisation of the solution to the problem for different specifications of preferences for both the Principal and the Agent. This is joint work with Camilo Hernàndez.

Mean Field Systems with Applications in Economics and Finance

Organizers : Fulvia Confortola, Andrea Cosso
Chairmans : Fulvia Confortola and Andrea Cosso

Huyên Pham
LPSM, Sorbonne University and Université de Paris
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Mean-field Markov decision process with common noise and randomized controls: convergence rate and applications to targeted advertising

We develop an exhaustive study of Markov decision process (MDP) under mean field interaction both on states and actions in the presence of common noise, and when optimization is performed over open-loop controls on infinite horizon. We highlight the crucial role of relaxed controls for this class of models, called CMKV-MDP for conditional McKean-Vlasov MDP, with respect to classical MDP theory. We prove the correspondence between CMKV-MDP and a general lifted MDP on the space of probability measures, and establish the dynamic programming Bellman fixed point equation satisfied by the value function, as well as the existence of ε-optimal randomized feedback controls. We obtain the propagation of chaos of the optimal value functions of the N-agent MDP to the CMKV-MDP when $N \to +\infty$, with some convergence rate, denoted by $O(M_N^\gamma)$. We finally provide examples of application of the propagation of chaos result, by approximately solving several toy models for N-agent targeted advertising problem with social influence via the resolution of the associated CMKV-MDP.
Based on joint work with Médéric Motte (LPSM).

Matteo Burzoni
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Mean field games with absorption and common noise with a model of bank run

We consider a mean field game describing the limit of a stochastic differential game of N-players whose state dynamics are subject to idiosyncratic and common noise and that can be absorbed when they hit a prescribed region of the state space. We provide a general result for the existence of weak mean field equilibria which, due to the absorption and the common noise, are given by random flow of sub-probabilities. We first use a fixed point argument to find solutions to the mean field problem

in a reduced setting resulting from a discretization procedure and then we prove convergence of such equilibria to the desired solution. We exploit these ideas also to construct ε-Nash equilibria for the N-player game. Since the approximation is two-fold, one given by the mean field limit and one given by the discretization, some suitable convergence results are needed. We also introduce and discuss a novel model of bank run that can be studied within this framework. This is a joint work with L. Campi.

Fausto Gozzi
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On mean field control and mean field games in infinite dimension

The aim of this talk is twofold. On one side, we report on recent work (with A. Cosso, I. Kharroubi, H. Pham, M. Rosestolato) on the optimal control of path-dependent McKean-Vlasov equations valued in Hilbert spaces. On the other side, we present the first ideas of a work, with S. Federico and M. Rosestolato, on mean field games in infinite dimension.

Most of the time will be devoted to the first topic, presenting some examples and the main results (the dynamic programming principle, the law invariance property of the value function V, Itô's formula and the fact that V is a viscosity solution of the HJB equation). We conclude presenting some motivating examples and the first ideas on the solution of the HJB-FKP system arising in mean field games in infinite dimension.

BSDEs in Credit and Default Risk

Organizer: Miryana Grigorova Chairwoman : Miryana Grigorova

Stéphane Crépey
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Two Machine Learning Numerical Schemes for a Class of Anticipated BSDEs

Motivated by the equations of cross valuation adjustments (XVAs) in the realistic case where capital is deemed fungible as a source of funding for variation margin, we study two numerical schemes for a class of anticipated BSDEs where the coefficient entails a conditional elicitable risk measure of the martingale part of the solution: an explicit scheme and an implicit scheme combined with a Picard iteration. We establish the time-consistency of both schemes and we provide elements for the analysis of the spatial regression error. A neural net regression implementation is benchmarked numerically in a high-dimensional and hybrid market/credit XVA use-case.

Marie-Claire Quenez
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European options in a non-linear incomplete market model with default

We study the superhedging prices and the associated superhedging strategies for European options in a nonlinear incomplete market model with default. The underlying market model consists of one risk-free asset and one risky asset, whose price may admit a jump at the default time. The portfolio processes follow nonlinear dynamics with a nonlinear driver f. By using a dynamic programming approach, we first provide a dual formulation of the seller's (superhedging) price for the European option as the supremum, over a suitable set of equivalent probability measures $Q \in \mathcal{Q}$, of the f - evaluation/expectation under Q of the payoff. We also establish a characterization of the seller's (superhedging) price as the initial value of the minimal supersolution of a constrained backward stochastic differential equation with default. Moreover, we provide some properties of the terminal profit made by the seller, and some results related to replication and no-arbitrage issues. Our results rely on first establishing a nonlinear optional and a nonlinear predictable decomposition for processes which are $\mathcal{E}^f$ -strong supermartingales under Q for all $Q \in \mathcal{Q}$.

[1] M., Grigorova, M.-C., Quenez, and A., Sulem *European options in a non-linear incomplete market model with default.* SIAM Journal on Financial Mathematics, Volume 11 (3), Pages 849–880, 2020.

Miryana Grigorova
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American options in a non-linear incomplete market model with default

In the framework of the *non-linear incomplete market models* introduced in the second talk of this session, we study the superhedging price and associated superhedging strategies for American options. We present a duality result for the seller's superhedging price, as well as an infinitesimal characterization in terms of a *Reflected* BSDE *with constraints* (under right-continuity assumption on the pay-off process of the option, as well as general results beyond this assumption). The talk is based on a joint work with Marie-Claire Quenez and Agnès Sulem.

[1] M., Grigorova, M.-C., Quenez, and A., Sulem *American options in a non-linear incomplete market model with default.* Stochastic Processes and their Applications, Volume 142 (12), Pages 479-512, 2021.

Wednesday

Contents

Pierre Cardaliaguet, 09:00 - 10:00 – Amphi 012

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Chairman : François Delarue

On the convergence rate for the optimal control of McKean-Vlasov dynamics

In this talk I will report on a joint work with S. Daudin (Paris Dauphine), Joe Jackson (U. Texas) and P. Souganidis (U. Chicago). We are interested in the convergence problem for the optimal control of McKean-Vlasov dynamics, also known as mean field control. We establish an algebraic rate of convergence of the value functions of N-particle stochastic control problems towards the value function of the corresponding McKean-Vlasov problem. This convergence rate is established in the presence of both idiosyncratic and common noise, and in a setting where the value function for the McKean-Vlasov problem need not be smooth. Our approach relies crucially on Lipschitz and semi-concavity estimates, uniform in N, for the N-particle value functions, as well as a certain concentration inequality. We also establish a quantified propagation of chaos property for the optimal solutions of the N-particule system for a dense subset of initial conditions.

Daniel Lacker, 10:00 - 10:30 – Amphi 012

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Chairman : François Delarue

Mean field approximations via log-concavity, and a non-asymptotic perspective on mean field control

We propose a new approach to deriving quantitative mean field approximations for log-concave probability measures on Euclidean space. The main application discussed in this talk is to a class of stochastic control problems in which a large number of players cooperatively choose their drifts to maximize an expected terminal reward minus a quadratic running cost. For a broad class of potentially asymmetric rewards, we show that there exist approximately optimal controls which are decentralized, in the sense that each player's control depends only on its own state and not the states of the other players. Moreover, the optimal decentralized controls can be constructed non-asymptotically, without reference to any mean field limit. The broader framework is inspired by the nonlinear large deviations theory of Chatterjee-Dembo, for which we offer an efficient new perspective in log-concave settings based on functional inequalities. Joint work with Sumit Mukherjee and Lane Chun Yeung.

Parallel sessions, 11:00 - 12:30

Parallel sessions take place in Amphi 012, Amphi 108, Room 125 and Room 126. Sessions are:

- **Mean Field Games and Mean Field Control** – Amphi 012
  Chairman: Pierre Cardaliaguet and François Delarue
- **Singular BSDEs** – Amphi 108
  Chairman: Alexandre Popier
- **From control theory to Deep Learning and back - 2** – Room 125
  Chairmen: David Siska and Lukasz Szpruch
- **Propagation of chaos** – Room 126
  Chairman: Daniel Lacker

Propagation of chaos

Contributed session
Chairman: Daniel Lacker

Khoa Le
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McKean–Vlasov equations with rough common noise and propagation of chaos

We show well-posedness and propagation of chaos for McKean–Vlasov equations with rough common noise and random adapted coefficients. Our results are valid under mild regularity assumptions on the coefficients which improve upon previous works. Our method relies on the previously established results for rough stochastic differential equations from Friz–Hocquet–Lê (2021+).

Benjamin Jourdain
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Central limit theorem over nonlinear functionals of empirical measures and applications to fluctuations of interacting particle systems

A generalised version of the central limit theorem is proposed for nonlinear functionals of the empirical measure of i.i.d. random variables, provided that the functional satisfies some regularity assumptions for the associated linear functional derivative. We use this result to deal with the contribution of the initialisation in the convergence of the fluctuations between the empirical measure of interacting diffusion and their mean-field limiting measure (as the number of particles goes to infinity). A complementary contribution related to the time evolution is treated using the master equation, a parabolic PDE involving L-derivatives with respect to the measure component, which is a stronger notion of derivative that is nonetheless related to the linear functional derivative.

Jean-François Jabir
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Propagation of chaos results via change of probability measure

This talk will be dedicated to present a recent approach to derive propagation of chaos properties, with optimal rates of convergence, for particle systems, related to McKean-Vlasov models, with interaction kernels not necessarily smooth but satisfying some concentration properties. The propagation of chaos will be mainly considered in the path-distribution sense, and rate of convergence evaluated in terms of the TV-distance and of Monge-Kantorovich-Wasserstein metrics on the path space. The derivation of our results essentially relies on a proper measurement of the Girsanov transform mapping the McKean-Vlasov dynamics and its particle approximation. Applications to some laboratory McKean-Vlasov models or more specific partially degenerated cases (e.g. Langevin-McKean-Vlasov models) will be presented, and a brief discussion on local-in-time weak propagation of chaos result will close the talk.

Singular BSDEs

Organizers: Guanxing Fu , Alexandre Popier , Chao Zhou
Chairman: Alexandre Popier

Yan Dolinsky
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Utility Maximization with Peeking into the Future

In this work we study optimal investment when the investor can peek some time unites into the future, but cannot fully take advantage of this knowledge because of quadratic transaction costs. In the Bachelier setting with exponential utility, we give an explicit solution to this control problem with intrinsically infinite-dimensional memory. This is made possible by solving the dual problem where we make use of the theory of Gaussian Volterra integral equations.

Devin Sezer
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Optimal Liquidation with Conditions on Minimum Price.

We consider the classical optimal liquidation problem where the goal of the investor is to close a position in a given time horizon and maximize expected utility of the terminal position. Typically these control problems are formulated with the constraint that the position is fully closed at terminal time. The goal of this research is to relax this last constraint and allow it to depend on the price of the underlying security. The main idea is to allow the trader to not be bound by the constraint if the price drops too much. We give several possible optimal control formulations of this problem and explore its solution via PDE and BSDE. For the BSDE the partial constraints lead to nondeterministic singular terminal conditions that can also take negative values. Another novelty is that the driver of the BSDE is no longer monotone but convex and the existing results in the literature are not directly applicable. Joint work with Mervan Aksu and Alexandre Popier.

Xiaonyu Xia
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Portfolio Liquidation Games with Self-Exciting Order Flow

We analyze novel portfolio liquidation games with self-exciting order flow. Both the N-player game and the mean-field game are considered. We assume that players' trading activities have an impact on the dynamics of future market order arrivals thereby generating an additional transient price impact. Given the strategies of her competitors each player solves a mean-field control problem. We characterize open-loop Nash equilibria in both games in terms of a novel mean-field FBSDE system with unknown terminal condition. Under a weak interaction condition we prove that the FBSDE systems have unique solutions. Using a novel sufficient maximum principle that does not require convexity of the cost function we finally prove that the solution of the FBSDE systems do indeed provide open-loop Nash equilibria. We will also talk about the liquidation problems when strategies are not continuous. We show that the value function as well as the optimal strategy can be characterized in terms of fully coupled ODE system. This is joint work with Ulrich Horst and Guanxing Fu.

From control theory to Deep Learning and back - 2

Organizers: David Šiška , Lukasz Szpruch
Chairman: Lukasz Szpruch

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Exploration vs Exploitation in Reinforcement Learning: Dilemma of the Controller in an Uncertain World

In stochastic control theory, the model is assumed to be known by the decision-makers; whereas, in Reinforcement Learning (RL), the decision-makers should infer the models and makes decisions through statistical estimates. In short, we may interpret the RL problem as a stochastic control problem, where the model uncertainty must be taken into consideration on top of the controller risk. This additional model uncertainty leads to the exploration-exploitation trade-off where the controller faces a dilemma between following the optimal control strategy obtained from the estimated model, and making the decision to understand the model more. In this talk, we will discuss such a phenomenon in the context of the linear-convex RL problem and propose a simple algorithm to balance such a trade-off obtained through the identifiability and sensitivity of the control problem.

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Entropic fictitious play for mean field optimization problem

It is well known that the training of the neural network can be viewed as a mean field optimization problem. In this paper we are inspired by the fictitious play, a classical algorithm in the game theory for learning the Nash equilibria, and propose a new algorithm, different from the conventional gradient-descent ones, to solve the mean field optimization. We rigorously prove its (exponential) convergence, and show some simple numerical examples.

Yufei Zhang
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Reinforcement learning for linear-convex models with jumps via stability analysis of feedback controls

We study finite-time horizon continuous-time linear-convex reinforcement learning problems in an episodic setting. In these problems, an unknown linear jump-diffusion process is controlled subject to nonsmooth convex costs. We start with the pure diffusion case with quadratic costs, and propose a least-squares algorithm which achieves a logarithmic regret bound of order $O((\log M)\,(\log\log M))$, with M being the number of learning episodes; the proof relies on the robustness of the associated Riccati differential equation and sub-exponential properties of the least-squares estimators. We then extend the least-squares algorithm to linear-convex learning problems with jumps, and establish a regret of the order $O((M \log M)^{1/2})$; the analysis leverages the Lipschitz stability of the associated forward-backward stochastic differential equation and concentration properties of sub-Weibull random variables.

This is joint work with Matteo Basei, Xin Guo and Anran Hu.

Mean field games and mean field control - 1

Organizers: Pierre Cardaliaguet, François Delarue
Chairman: Pierre Cardaliaguet

Idris Kharroubi
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Optimal control of path-dependent McKean-Vlasov SDEs in infinite dimension

We study the optimal control of path-dependent McKean-Vlasov equations valued in Hilbert spaces motivated by non Markovian mean-field models driven by stochastic PDEs. We first establish the well-posedness of the state equation, and then we prove the dynamic programming principle (DPP) in such a general framework. The crucial law invariance property of the value function V is rigorously obtained, which means that V can be viewed as a function on the Wasserstein space of probability measures on the set of continuous functions valued in Hilbert space. We then define a notion of pathwise measure derivative, which extends the Wasserstein derivative due to Lions [Cours au collège de france: Théorie des jeux à champ moyens. Audio Conference, 2006-2012], and prove a related functional Itô formula in the spirit of Dupire [Functional itô calculus. SSRN: 1435551, 2009] and Wu and Zhang [Viscosity solutions to parabolic master equations and McKean-Vlasov SDEs with closed-loop controls. Ann. Appl. Probab., 30(2):936-986, 2020]. The Master Bellman equation is derived from the DPP by means of a suitable notion of viscosity solution. We provide different formulations and simplifications of such a Bellman equation notably in the special case when there is no dependence on the law of the control. This talk is based on a joint work with Andrea Cosso, Fausto Gozzi, Huyên Pham and Mauro Rosestolato.

Giulia Livieri
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N-player games and mean field games of moderate interactions

In this joint work with Franco Flandoli and Maddalena Ghio, we study the asymptotic organization among many optimizing individuals interacting in a suitable “moderate” way. We justify this limiting game by proving that its solution provides approximate Nash equilibria for large but finite player games. This proof depends upon the derivation of a law of large numbers for the empirical processes in the limit as the number of players tends to infinity. Because it is of independent interest, we prove this result in full detail. We characterize the solutions of the limiting game via a verification argument.

Chenchen Mou
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Propagation of Monotonicity for Mean Field Game Master Equations.

It is well known in the mean field game literature that certain monotonicity condition is crucial for the uniqueness of mean field equilibria and for the wellposedness of the associated master equation. One interesting observation is that the propagation of the monotonicity (either in Lasry-Lions sense or in displacement sense) of the value function plays the key role here. We shall introduce a method to find conditions on the coefficients which ensure that any solution of the master equation will maintain the monotonicity property. This method also allows us to consider anti-monotonicity and obtain the desired wellposedness provided the coefficients are sufficient anti-monotone in appropriate sense. We finally extend our results to mean field game of controls. This talk is based on a joint work with Gangbo-Meszaros-Zhang and two joint works with Zhang.

Thursday

Contents

Jin Ma, 09:00 - 10:00 – Amphi 012

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Chairman : Jiongmin Yong

On Set-valued Backward SDEs and Related Set-valued Stochastic Analysis

We try to establish an analytic framework for studying Set-Valued Backward Stochastic Differential Equations (SVBSDE for short), motivated largely by the current studies of dynamic set-valued risk measures for multi-asset or network-based financial models. Adopting the notion of Hukuhara difference between sets, which compensates the lack of "inverse" operation of the traditional Minkowski addition, whence the vector space structure, in traditional set-valued analysis, we first examine and establish a useful foundation of set-valued stochastic analysis, and identify some fundamental challenges regarding Aumann-Itô integrals, especially when it is connected to the martingale representation theorem. We shall then propose some extensions of the existing theory that are necessary to study the SVBSDEs, including a new framework of a Pettis-type stochastic analysis and stochastic integrals that would hopefully resolve the fundamental conflicts occurred in the current theory of Aumann-Itô stochastic integrals.

This talk is based on the joint works with Çagin Ararat (Bilkent University, Turkey).

Rainer Buckdahn, 10:30 - 11:00 – Amphi 012

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Chairman : Jiongmin Yong

Mean-Field BDSDEs and associated nonlocal semi-linear backward SPDEs

In this joint work with Juan Li and Chuanzhi Xing (Shandong University, Weihai, China) we investigate mean-field BDSDEs, i.e., BDSDEs whose driving coefficients depend not only on the solution process (Y, Z) and the solution X of an associated mean-field forward-SDE but also on their joint law. Given the initial condition $(t, x, P_\xi) \in [0, T] \times R^n \times \mathcal{P}_2(R^n)$, we prove the L^2-regularity of these solution processes $(X^{t,x,P_\xi}, Y^{t,x,P_\xi}, Z^{t,x,P_\xi})$ with respect to the initial condition as well as that of the value function $V(t, x, P_\xi) = Y_t^{t,x,P_\xi}$. However, unlike the pioneering paper on BDSDEs by Pardoux and Peng, as $\mathcal{P}_2(R^n)$ is infinite-dimensional, we cannot use the argument of Kolmogorov's continuity criterion to get C^2 regularity with respect to (x, P_ξ). But the observation that, for all $\eta \in L^\infty(\mathcal{F}; R)$, the function $\Psi(t, x, P_\xi) = E[V(t, x, P_\xi)\eta]$ is a $C^{1,2,2}$ function allows to characterise V is a unique (nearly) classical solution of an associated backward SDE.

As a byproduct we extend the classical mean-field Itô formula to smooth functions of solutions of mean-field BDSDEs.

Jiongmin Yong, 11:00 - 11:30 – Amphi 012

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Chairman : Rainer Buckdahn

Forward-Backward Stochastic Differential Equations: Initiation, Development and Beyond

Jin Ma has made fundamental contributions to the theory of forward-backward stochastic differential equations (FBSDEs, for short). In this talk, as one of his main collaborators, I will recall the initiation, the path of the development for FBSDEs, as well as some interesting historic stories.

Jianfeng Zhang, 11:30 - 12:00 – Amphi 012 (remote talk)

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Chairman : Rainer Buckdahn

Set Valued HJB Equations

The set values have been introduced for many applications, such as time inconsistent stochastic optimization problems, multivariate dynamic risk measures, and nonzero sum games with multiple equilibria. Among others, one crucial property of the dynamic set value is the dynamic programming principle. In this talk we introduce a notion of set valued PDEs and show that the set value function of certain multidimensional control problem is the unique solution to the corresponding set valued HJB equation. A key tool is the set valued Ito formula, which together with the DPP induces the PDE. In the one dimensional case, our set valued PDE reduces back to the standard HJB equation.

Our characterization of the set values is through their boundaries, which are manifolds. Thus our approach is intrinsically connected to the existing theory of moving/evolving manifolds, such as front propagation and mean curvature flows. Roughly speaking, those equations can be viewed as first order set valued ODEs, and we extend them to second order PDEs. Another difference is that, due to different applications, those equations are forward in time (with initial conditions), while we consider backward equations (with terminal conditions).

Parallel sessions, 13:30 - 15:30

Parallel sessions take place in Amphi 012, Amphi 108, Room 125 and Room 126. Sessions are:

- **Recent advances in continuous time principal-agent problem** – Amphi 012
  Chairman: Xiaolu Tan
- **Recent advances in martingale representation theorems and enlarged filtrations** – Amphi 108
  Chairman: Claudio Fontana
- **Mean-Field Systems and applications - 2** – Room 125
  Chairman: Jean-François Mehdi Jabir
- **Mean-Field Systems and applications - 3** – Room 126
  Chairman: Pierre Cardaliaguet

Recent advances in dynamic principal-agent problem

Organizers : Yiqing Lin and Xiaolu Tan
Chairman : Xiaolu Tan

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Moral hazard for time-inconsistent agents and BSVIEs

We address the problem of Moral Hazard in continuous time between a Principal and an Agent that has time-inconsistent preferences. Building upon our previous results on non-Markovian time-inconsistent control for sophisticated agents, we are able to reduce the problem of the principal to a novel class of control problems exposing the hardships brought upon the problem by the agent's preferences. The structure of this problem comes from the representation of the problem of the Agent via a so-called extended Backward Stochastic Volterra Integral equation. We will present some results on the characterization of the solution to problem for different specifications of preferences for both the Principal and the Agent. This talk covers joint work with Dylan Possamaï (ETH).

Hao Xing
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Milestone Bonus

We develop a dynamic principal-agent model for financing a multi-stage project which does not generate intermediate cash flow but produces a lump sum payoff when the final stage succeeds. We show that, when the intermediate milestones are successfully completed, principal prefers to reward agent with deferred compensations rather than milestone bonus. Agent's equity stake decreases and becomes less sensitive to agent's performance in later stages. When principal observes a signal about agent's action, contracting on signal and success reward substitute each other in incentive provision. When negative evidence against agent's effort accumulates, principal utilises more precise signal, higher contract sensitivity, and reduces the success reward. This is a joint work with Kerry Back and Ali Kakhbod.

Junjian Yang
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Path-dependent mean-field game optimal planning

In the context of mean-field games, with possible control of the diffusion coefficient, we consider a path-dependent version of the planning problem introduced by P.L. Lions: given a pair of marginal distributions (μ_0, μ_1), find a specification of the game problem starting from the initial distribution μ_0, and inducing the target distribution μ_1 at the mean-field game equilibrium. Our main result reduces the path-dependent planning problem into an embedding problem, that is, constructing a McKean-Vlasov dynamics with given marginals (μ_0, μ_1). Some sufficient conditions on (μ_0, μ_1) are provided to guarantee the existence of solutions. We also characterize, up to integrability, the minimum entropy solution of the planning problem. In particular, as uniqueness does not hold anymore in our path-dependent setting, one can naturally introduce an optimal planning problem which would be reduced to an optimal transport problem along with controlled McKean-Vlasov dynamics.

Recent advances in martingale representation and enlargement of filtrations

Organizer : Claudio Fontana
Chairman : Claudio Fontana

Anna Aksamit
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Information modelling: new type of filtration enlargement, representation property and applications

In this talk we will review the classical results about enlargement of filtration and present some new development with applications. The focus will be on conditions under which martingales in the reference filtration remain semimartingales in the large filtration, in which case, the canonical decomposition is of particular interest. We will then present enlargement of a reference filtration through the observation of a random time and a mark. Random time considered is such that its graph is included in the countable union of graphs of stopping times. Mark revealed at this random time is assumed to satisfy generalised Jacod's condition. Our relaxation of Jacod's condition accounts for the dynamic structure of the problem. Finally we will discuss stability of martingale representation property and further application. Talk is based on a joint work with Claudio Fontana.

Barbara Torti
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Martingale representations in progressive enlargement by multivariate point processes

Let us consider a finite set of multivariate point processes $\{(T_n^i, X_n^i)\}_{n\geq 1}$, $i = 1, \dots, d$, defined on the same probability space and with explosion time taking values in $(0, +\infty]$ and let $\mathbb{G}$ be the initially enlarged natural filtration of the vector process $\{(T_n^i, X_n^i)\}_{n\geq 1}$, $i = 1, \dots, d$. We discuss weak and strong representations of all $\mathbb{G}$-local martingales. In particular, we prove that the local martingales are weakly represented up to the minimum among the explosion times. We also prove that a strong representation holds if any multivariate point process has almost surely infinite explosion time and discrete mark's space. Finally, we give a condition which assures the components of the multidimensional local martingale driving the strong representation to be pairwise orthogonal. This talk is based on a joint work with Antonella Calzolari.

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On the propagation of the weak representation property in progressively enlarged filtrations

An $\mathbb{F}$-adapted semimartingale $(X, \mathbb{F})$ is said to possess the weak representation property (from now on WRP) with respect to a filtration $\mathbb{F}$ if every $\mathbb{F}$-local martingale can be represented as a stochastic integral of a predictable process with respect to the continuous local martingale part X^c of X plus a stochastic integral of a predictable function with respect to the compensated jump measure $\mu^X - \nu^X$ of X. A well-known example in which the WRP holds is if X is a Lévy process and $\mathbb{F}$ is generated by X.

In this talk we consider a semimartingale $(X, \mathbb{F})$ possessing the WRP with respect to the reference filtration $\mathbb{F}$ and we enlarge $\mathbb{F}$ progressively by another filtration $\mathbb{H}$, that is, we consider the smallest right-continuous filtration $\mathbb{G}$ containing both $\mathbb{F}$ and $\mathbb{H}$. We then state some sufficient conditions on $\mathbb{F}$ and $\mathbb{H}$ to ensure that $\mathbb{G}$ inherits the WRP of X with respect to $\mathbb{F}$. We consider the case in which $\mathbb{G}$ is obtained enlarging $\mathbb{F}$ by a random time τ (meaning that $\mathbb{G}$ is the smallest filtration containing $\mathbb{F}$ and such that τ is a stopping time) but also more general situations in which $\mathbb{F}$ is enlarged by a whole semimartingale, that is, $\mathbb{H}$ is generated by another semimartingale Y, which may possess the WRP with respect to $\mathbb{H}$. We do not exclude that X and Y have simultaneous jumps and we do not require that they are quasi-left continuous.

Jasmina Dordevic
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Reflected backward stochastic differential equations with time-change Levy noises

We study reflected backward stochastic differential equations with a left/lower barrier (RBSDEs with lower barrier) for time-changed Levy noises when the time-change is independent of the Levy process. Existence and uniqueness of the solution of RBSDE with left barrier under natural filtration F and under enlarged filtration G which contains additional informations with respect to time-change process are proved. Further, comparison principle for this equation is obtained.

This is join work with Prof. Giulia Di Nunno.

Mean Field Systems and Applications - 2

Contributed Session
Chairman: Jean-François Mehdi Jabir

Mohamed Bahlali
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A mean-field approach for the spatial distribution of labour, housing and urban air pollution

There exists a relationship between urban air pollution and economic activity: economic activity generates pollution, for instance through heating and transportation ; in turn, pollution spreads around and generates economic disutility. We develop a mean-field model of city coupling a labour market, a housing market, and pollution resulting from automobile commuting, which is modelled through an diffusion equation aiming at representing its physical dispersion. Agents choose where to work and live in order to maximize their utility, by consuming goods, housing surface and by avoiding pollution. We prove existence of equilibria, and uniqueness when the number of job locations is finite. We provide numerical simulations and we obtain analytical results in the case of a linear monocentric city.

Maximilien Germain
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A level-set approach to the control of state-constrained McKean-Vlasov equations: application to renewable energy storage and portfolio selection

We consider the control of McKean-Vlasov dynamics (or mean-field control) with probabilistic state constraints. We rely on a level-set approach which provides a representation of the constrained problem in terms of an unconstrained one with exact penalization and running maximum or integral cost. The method is then extended to the common noise setting. Our work extends (Bokanowski, Picarelli, and Zidani, SIAM J. Control Optim. 54.5 (2016), pp. 2568–2593) and (Bokanowski, Picarelli, and Zidani, Appl. Math. Optim. 71 (2015), pp. 125–163) to a mean-field setting. The reformulation as an unconstrained problem is particularly suitable for the numerical resolution of the problem, that is achieved from an extension of a machine learning algorithm from (Carmona, Laurière, arXiv:1908.01613 to appear in Ann. Appl. Prob., 2022). A first application concerns the storage of renewable electricity in the presence of mean-field price impact and another one focuses on a mean-variance portfolio selection problem with probabilistic constraints on the wealth. We also illustrate our

approach for a direct numerical resolution of the primal Markowitz continuous-time problem without relying on duality.

Alexander Vogler
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Optimal control of mean field equations with monotone coefficients and applications in neuroscience

We are interested in the optimal control problem associated with certain quadratic cost functionals depending on the solution $X = X^\alpha$ of the stochastic mean-field type evolution equation in $\mathbb{R}^d$

$$dX_t = b(t, X_t, \mathcal{L}(X_t), \alpha_t)dt + \sigma(t, X_t, \mathcal{L}(X_t), \alpha_t)dW_t\,, \quad X_0 \sim \mu \;\; (\mu \text{ given}), \tag{4.1}$$

under assumptions that enclose a sytem of FitzHugh-Nagumo neuron networks, and where for practical purposes the control α_t is deterministic. To do so, we assume that we are given a drift coefficient that satisfies a one-sided Lipshitz condition, and that the dynamics (4.1) satisfy an almost sure boundness property of the form $\pi(X_t) \leq 0$. The mathematical treatment we propose follows the lines of the recent monograph of Carmona and Delarue for similar control problems with Lipschitz coefficients. After addressing the existence of minimizers via a martingale approach, we show a maximum principle for (4.1), and numerically investigate a gradient algorithm for the approximation of the optimal control.

Mean Field Systems and Applications - 3

Contributed Session
Chairman: Pierre Cardaliaguet

Robert Denkert
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Mean Field Control Problems with Singular Controls

We consider a novel class of mean field control (MFC) problems with singular controls. The costs dependent on the current state, control and the joint law of the state-control-process. We derive a dynamic programming principle and use this to derive a quasi-variational inequality (QVI) for the value function in the Wasserstein space. Subsequently, we prove an approximation of general singular controls with purely regular controls. Finally, we use the master equations of the approximating regular MFC problems to establish a uniqueness result for our QVI characterisation of the value function of the MFC problem with singular control.

Julian Wendt
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Large ranking games with diffusion control

In this talk we consider a symmetric stochastic differential game where each player can control the diffusion intensity of an individual dynamic state process, and the players whose states at a deterministic finite time horizon are among the best $\alpha \in (0, 1)$ of all states receive a fixed prize. Within the mean field limit version of the game we compute an explicit equilibrium, a threshold strategy that consists in choosing the maximal fluctuation intensity when the state is below a given threshold, and the minimal intensity else. We show that for large n the symmetric n-tuple of the threshold strategy provides an approximate Nash equilibrium of the n-player game. We also derive the rate at which the approximate equilibrium reward and the best response reward converge to each other, as the number of players n tends to infinity. Finally, we compare the approximate equilibrium for large games with the equilibrium of the two player case.

Samuel Daudin
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Mean Field Control with state constraints in the Wasserstein Space

We consider an optimal control problem for the Fokker-Planck equation subject to a state constraint in the space of probability measures equipped with the L^2-Wasserstein distance. The first order necessary optimality conditions associated to this optimization problem give rise to a second order mean field game (MFG) system which has a potential structure. Since the problem is subject to the state constraint, additional unknowns (beside the usual density and value function variables) appear in the MFG system, which can be seen as the corresponding Lagrange multipliers. We will explain how to derive the optimality conditions and what we can deduce for the corresponding solutions. In particular we will show that -despite the presence of the state constraint- optimal controls are Lipschitz continuous in time.

Shahlar Meherrem
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Necessary optimality Conditions for Stochastic Mean Field Control System

In this presentation, first we consider a general characterization of the optimal stochastic combined control for mean-field jump-systems which is derived by applying mixed convex-spike perturbation method. The diffusion coefficient depends on the continuous control variable and the control domain is not necessary convex. In our combined mean-field control problem, we discuss two classes of jumps for the state processes, the inaccessible jumps which caused by Poisson martingale measure and the predictable ones which caused by the singularity of the control variable. Markowitz's mean-variance portfolio selection problem with intervention control is discussed as an application.

Secondly, the work to be discussed is as follows: Stochastic singular control for mean-field forward-backward stochastic differential equations, driven by orthogonal Teugels martingales associated with some Levy processes heaving moments of all orders and an independent Brownian motion. Under partial information, necessary and sufficient conditions for optimality in the form of maximum principle for this mean-field system are established by means of convex variation methods and duality techniques. As an illustration, this paper studies a partial information mean-variance portfolio selection problem driven by orthogonal Teugels martingales associated with Gamma process as Levy process of bounded variation.

Main work of this presentation is to be considered as: Necessary and sufficient conditions for optimal singular control for systems governed by general controlled McKean-Vlasov differential equations, in which the coefficients depend on the state of the solution process as well as of its law and the control. The control domain is assumed to be convex. The control variable has two components, the first being absolutely continuous and the second singular. The proof of our result is based on the derivative of the solution process with respect to the probability law and a corresponding Ito formula. Finally, an example is given to illustrate the theoretical results.

Marcel Nutz, 16:00 - 16:30 – Amphi 012 (remote talk)

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Chairman : Dylan Possamaï

Entropic Optimal Transport and Convergence of Sinkhorn's Algorithm

Applied optimal transport is flourishing after computational advances have enabled its use in real-world problems with large data sets. Entropic regularization is a key method to approximate optimal transport in high dimensions while retaining feasible computational complexity. In this talk we discuss entropic optimal transport and its stability with respect to the marginals. A qualitative result (for weak convergence) is obtained using the geometric notion of c-cyclical invariance and a quantitative result (for Wasserstein distance) is obtained by control theoretic methods. These results can be applied to deduce convergence of Sinkhorn's algorithm for unbounded cost functions such as the quadratic cost and find a convergence rate in Wasserstein sense. Based on joint works with Espen Bernton, Stephan Eckstein, Promit Ghosal, Johannes Wiesel.

Parallel sessions, 16:45 - 18:15

Parallel sessions take place in Amphi 012, Amphi 108, Room 125 and Room 126. Sessions are:

- **Backward Stochastic Partial Differential Equations and Control Theory of Stochastic Partial Differential Equations** – Amphi 012
  Chairman: Qi Lü
- **Optimal switching or stopping control problems** – Amphi 108
  Chairman: Adrien Richou
- **Mean Field Games and Mean Field control** – Room 125
  Chairmen: Pierre Cardaliaguet and François Delarue
- **BSDEs in infinite dimensional control** – Room 126
  Chairman: Giuseppina Guatteri

Backward Stochastic Partial Differential Equations and Control Theory of Stochastic Partial Differential Equations

Organizer : Qi Lü
Chairman: Qi Lü

Xu Liu
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Carleman Estimates for Some Stochastic Partial Differential Equations

Carleman-type estimate is one of useful tools in studying control and inverse problems for deterministic/stochastic PDEs. In this talk, the weighed identity method and duality technique twice are introduced to establish stochastic Carleman estimates. These two methods are also compared with each other. As applications, the observability and controllability problems for some stochastic PDEs are studied, respectively.

Qi Lü
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Applications of Backward SPDEs in Control Theory of SPDEs

Backward stochastic partial differential equations play key role in the study of many control problems for stochastic partial differential equations. In this talk, I will present some typical examples from controllability and optimal control problems of stochastic partial differential equations.

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Error Estimates for Space-Time Discretization of a Linear-Quadratic Control Problem for the Stochastic Heat Equation

In this talk, we introduce a time-implicit, finite-element based space-time discretization scheme of a linear-quadratic control problem governed by the stochastic heat equation, and this discretization scheme is essential to approximate a coupled forward-backward stochastic heat equation. The above fully discrete version is then solved by a gradient descent algorithm. For this scheme, convergence rate is proved. This is a joint work with Prof. Andreas Prohl (University of Tuebingen).

Optimal switching or stopping control problems

Organizer : Adrien Richou
Chairman : Adrien Richou

Cyril Bénézet
ENSIIE - Laboratoire de Mathématiques et Modélisation d'Évry (LaMME)
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Switching problems with controlled randomisation

In this work, we introduce and study a new class of optimal switching problems, where the agent does not choose directly the new mode, but rather a probability distribution under which the new mode is chosen. We show, in the spirit of Hu and Tang (2010), when the switching costs are positive, that the value of these problems and an optimal policy are obtained by solving obliquely reflected BSDEs. We then study, with arbitrary costs and in what we call the uncontrolled irreducible case, the question of existence of solutions of these obliquely reflected BSDEs in a Markovian framework. This is a joint work with J.-F. Chassagneux and A. Richou.

Tiziano De Angelis
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Mean-Field Games of Finite-Fuel Capacity Expansion with Singular Controls.

We study Nash equilibria for a sequence of symmetric N-player stochastic games of finite-fuel capacity expansion with singular controls and their mean-field game counterpart. We construct a solution of the MFG via a simple iterative scheme that produces an optimal control in terms of a Skorokhod reflection at a (state-dependent) surface that splits the state space into action and inaction regions. We then show that a solution of the MFG of capacity expansion induces approximate Nash equilibria for the N-player games with approximation error ε going to zero as N tends to infinity. Our analysis relies entirely on probabilistic methods and extends the well-known connection between singular stochastic control and optimal stopping to a mean-field framework. This is a joint work with L. Campi, M. Ghio, G. Livieri.

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Optimal switching with an infinite set of modes: An approach by randomization & constrained BSDEs

In this talk, a brief presentation of both the primal and dual stochastic optimal switching problems will be given. In the context of an infinite number of modes and since the study is done in a Non Markovian setting, this requires to define properly the probabilistic set-up (both for the primal and dual control problem) as well as the notion of admissible control strategies for each control problems. In particular, we shall explain in details the construction of the dual control problem which is based on the so-called randomization approach (we mention that this method has been successfully used by Pr Marco Fuhrman and co-authors in recent papers to study several general stochastic control problems).

The two main results are the following ones: the equality between the two value functions: the representation of the dual value function in terms of the minimal solution of a Backward differential equations with constraints on the jump term.

If time allows, a brief sketch of one of the two inequalities shall be given (the first result being the central result of the paper). Some perspectives of the present work shall be discussed. This is a joint work with Pr Marco Furhman (Universita degli Studi di Milano Italy).

Mean field games and mean field control - 2

Organizers : Pierre Cardaliaguet, François Delarue
Chairman : François Delarue

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Mean field game master equations: from finite to continuous state space.

We analyze mean field games for diffusion-based models and their space discretization. For simplicity, the dynamics evolves on the one-dimensional torus, and there is no common noise. We examine a space discretization of the dynamics, which is reminiscent of the Markov chain approximation method in stochasctic optimal control, but also of finite difference numerical schemes for Hamilton-Jacobi equations. The discretized problem turns out to be a mean field game over a finite state space, still in continuous time. Assuming monotonicity, which ensures uniqueness and also stability, we show that the discrete mean field games converge to the continuous limit, the main result being to provide a rate for the convergence both of the master equations governing the games and of the optimal trajectories. We consider first the case in which there exists a smooth solution to the limit master equation and then the case where there is no such solution. In the end, we consider briefly the case of common noise and show how the notion of monotone solutions (introduced by C. Bertucci) helps in proving the convergence of the master equations, by means of compactness arguments. Based on joint work with Charles Bertucci (CNRS - Ecole polytechnique)

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Mean Field Game of Mutual Holding

In this presentation, we will talk about a mean field model for optimal holding of a representative agent of her peers as a natural expected scaling limit from the corresponding N?agent model. The induced mean field dynamics appear in a form which is not covered by standard MFG framework. We will start by briefly discussing the common noise case, which has the particularity of taking into account the control of the diffusion coefficient. This feature is less common in MFG framework. Then, we will study the corresponding mean field game of mutual holding in the absence of common noise. Our first main result provides an explicit equilibrium of this mean field game, defined by a bang?bang control consisting in holding those competitors with positive drift coefficient of their dynamic value. We will next use this mean field game equilibrium to construct (approximate) Nash equilibria for the

corresponding N?player game. We also provide some numerical illustrations of our mean field game equilibrium which highlight some unexpected effects induced by our results. This is a joint work with Nizar Touzi.

Emma Hubert
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Electricity demand response: a mean-field contract theory approach

In this talk, we introduce a problem of contract theory, in continuous-time, involving a continuum of agents with mean-field interactions. The incentive model we formulate, where a principal can contract with an infinite number of agents, generalizes the one-agent framework by Aïd, Possamaï and Touzi [1]. The motivation behind this extension is to improve the results obtained for the application considered in the previous paper, *i.e.* the electricity demand response program. More precisely, the principal (an electricity producer, or provider) contracts with an agent (a consumer), to incentivise him to decrease the mean and the volatility of his energy consumption during high peak demand. The mean-field formulation is justified by the fact that the electricity producer/provider is facing a large number of consumers, and should optimise the total consumption, not only the consumption of a particular agent. In addition, we introduce a common noise, impacting all consumption processes, in order to represent the common random environment in which consumers evolve. Indeed, the weather conditions have a significant impact on the agents's electricity consumption. Therefore, the principal continuously observes the consumption of a continuum of risk-averse consumers, but not the efforts they make in the different usages, and designs contracts in order to reduce her production costs. This mean-field framework leads us to consider a more extensive class of contracts. In particular, we prove that the producer can benefit from considering the mean-field of consumers by indexing contracts on the consumption of one agent and aggregate consumption statistics from the distribution of the entire population of consumers. The proof that this new form of contracts is without loss of generality relies on second-order backward stochastic differential equations (2BSDEs) of a mean-field type. We illustrate the results numerically.

Based on the joint work [2] with Romuald Élie, Thibaut Mastrolia and Dylan Possamaï.

[1] Aïd, René, Possamaï, Dylan and Touzi, Nizar *Optimal electricity demand response contracting with responsiveness incentives*. Mathematics of Operations Research, 2022.

[2] Elie, Romuald, Hubert, Emma, Mastrolia, Thibault and Possamaï, Dylan. *Mean-field moral hazard for optimal energy demand response management*. Mathematical Finance, 31(1), 2021.

BSDEs in infinite dimensional control

Organizer : Giuseppina Guatteri
Chairwoman : Giuseppina Guatteri

Federica Masiero
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A BSDEs approach to pathwise uniquenes for stochastic evolution equations

We prove strong well-posedness for a class of stochastic evolution equations in Hilbert spaces H when the drift term is Hölder continuous. This class includes examples of semilinear stochastic (damped) wave equations which describe elastic systems. (for such equations existence of solutions in the linear case is also a delicate issue) and semilinear stochastic 3D heat equations. In the deterministic case, there are examples of non-uniqueness in our framework. Strong (or pathwise) uniqueness is restored by means of a suitable additive Wiener noise. The proof of uniqueness relies on the study of related systems of infinite dimensional forward-backward SDEs (FBSDEs). This is a different approach with respect to the well-known method based on the Itô formula and the associated Kolmogorov equation (the so-called Zvonkin transformation or Itô-Tanaka trick). .

Luca Scarpa
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Forward-backward stochastic systems via convex minimisation

We present the Weighted Energy–Dissipation (WED) variational principle for stochastic evolution equations in Hilbert spaces. The approach consists in minimising suitable convex cost functionals, defined on spaces of entire trajectories, and depending on an approximation parameter. The corresponding Euler–Lagrange equation yielding first-order conditions for optimality is characterised as a forward–backward nonlinear stochastic evolution system. This can be equivalently seen as an elliptic-in-time regularisation of the original stochastic equation with respect to the approximation parameter. This study is based on the joint work [1] with Prof. U. Stefanelli (University of Vienna, Austria).

[1] Ulisse, Stefanelli and Luca, Scarpa. *Stochastic PDEs via convex minimization.* Communications in Partial Differential Equations, 2021

Gianmario Tessitore
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Singular Limit of Two Scale Stochastic Optimal Control Problems in Infinite Dimensions by Vanishing Noise Regularization

We study the limit of the value function for a two-scale, infinite-dimensional, stochastic controlled system with cylindrical noise and possibly degenerate diffusion. The limit is represented as the value function of a new reduced control problem (on a reduced state space). The presence of a cylindrical noise prevents representation of the limit by viscosity solutions of HJB equations, while degeneracy of diffusion coefficients prevents representation as a classical BSDE. We use a vanishing noise regularization technique.

Friday

Contents

Talk of Thaleia Zariphopoulou, 09:00 - 10:00

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Chairman : Marco Fuhrman

Mean field games in Ito-diffusion markets under general preferences

Chairman : Marco Fuhrman In my talk, I will introduce n-player and mean field games in Ito-diffusion markets with relative performance criteria. I will discuss their solvability and present closed form solutions for general payoffs, as well as examples in the class of completely monotonic inverse marginal utilities.

Parallel sessions, 10:30 - 12:00

Parallel sessions take place in Amphi 012, Amphi 108, Room 125 and Room 126. Sessions are:

- **Nonlinear Expectations** – Amphi 012 (remote session)
  Chairman: Mingshang Hu
- **Mean-Field Reflected Backward Stochastic Differential Equations** – Amphi 108
  Chairman: Saïd Hamadène
- **Numerical methods for non-linear McKean-Vlasov SDEs** – Room 125
  Chairman: Goncalo Dos Reis
- **Connection with PDEs - 3** – Room 126
  Chairman: Elena Issoglio

Nonlinear Expectations

Organizers : Mingshang Hu
Chairman: Mingshang Hu

Shaolin Ji
Shandong University, China
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A global stochastic maximum principle for fully coupled forward-backward stochastic systems

We study a stochastic optimal control problem for fully coupled forward-backward stochastic control systems with a nonempty control domain. For our problem, the first-order and second-order variational equations are fully coupled linear FBSDEs. Inspired by Hu [1], we develop a new decoupling approach by introducing an adjoint equation which is a quadratic BSDE. By revealing the relations among the terms of the first-order Taylor's expansions, we estimate the orders of them and derive a global stochastic maximum principle which includes a completely new term. Applications to stochastic linear quadratic control problems are investigated.

[1] M. Hu, *Stochastic global maximum principle for optimization with recursive utilities.* Probab. Uncertain. Quant. Risk 2(1) (2017) 1-20.

[2] M. Hu, S. Ji and X. Xue, *A global stochastic maximum principle for fully coupled forward-backward stochastic systems.* SIAM J. CONTROL OPTIM. 56(6) (2018) 4309-4335.

Yongsheng Song
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Stein's Method under Sublinear Expectations

In this talk, we introduce Stein's method under sublinear expectations and its applications to error estimates for Peng's central limit theorem and law of large numbers under sublinear expectations.

[1] M. Hu, S. Peng and Y. Song, *Stein type characterization for G-normal distributions.* Electron. Commun. Probab. 22(24) (2017) 1-12.

[2] X. Fang, S. Peng, Q. Shao and Y. Song, *Limit theorems with rate of convergence under sublinear expectations.* Bernoulli 25(4A) (2019) 2564-2596.

[3] Y. Song, *Normal approximation by Stein's method under sublinear expectations.* Stochastic Process. Appl. 130 (2020) 2838-2850.

[4] Y. Song, *Stein's method for law of large numbers under sublinear expectations.* Probab. Uncertain. Quant. Risk 6(3) (2021) 199-212.

Mingshang Hu
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Dynamic programming principle for stochastic recursive optimal control problem driven by a G-Brownian motion 3

We study a stochastic recursive optimal control problem in which the cost functional is described by the solution of a backward stochastic differential equation driven by G-Brownian motion. Under standard assumptions, we establish the dynamic programming principle and the related fully nonlinear HJB equation in the framework of G-expectation. Finally, we show that the value function is the viscosity solution of the obtained HJB equation.

[1]M. Hu, S. Ji, S. Peng and Y. Song, *Backward stochastic differential equations driven by G-Brownian motion.* Stochastic Process. Appl. 124 (2014) 759-784.

[2] M. Hu, F. Wang and G. Zheng, *Quasi-continuous random variables and processes under the G-expectation framework.* Stochastic Process. Appl. 126 (2016) 2367-2387.

[3]M. Hu and S. Ji, *Dynamic programming principle for stochastic recursive optimal control problem driven by a G-Brownian motion.* Stochastic Process. Appl. 127 (2017) 107-134.

Mean-Field Reflected Backward Stochastic Differential Equations

Organizer: Saïd Hamadene
Chairman: Saïd Hamadene

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BSDEs with weak reflections and partial hedging of American options

We introduce a new class of Backward Stochastic Differential Equations with weak reflections whose solution (Y, Z) satisfies the weak constraint $E[\Psi(\theta, Y_\theta)] \geq m$, for all stopping time θ taking values in $[0, T]$, where Ψ is a random non-decreasing map and m a given threshold. We study the well-posedness of such equations and show that the family of minimal time-t-values Y_t (where (Y, Z) is a supersolution of the BSDE with weak reflections) can be aggregated by a right continuous process. We give a nonlinear Mertens type decomposition for lower reflected g-submartingales, which to the best of our knowledge, represents a new result in the literature. Using this decomposition, we obtain a representation of the process of minimal time-t-values Y_t. Moreover, we show that this process can be written as a stochastic control/optimal stopping game, which is shown to admit, under appropriate assumptions, a value and saddle points. From a financial point of view, this problem is related to the approximative hedging for American options.

[1] Dumitrescu, R., Elie, R., Sabbagh, W., and Zhou, C. *BSDEs with weak reflections and partial hedging of American options.* arXiv preprint arXiv:1708.05957, 2017.

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A propagation of chaos result for a class of mean-field reflected BSDEs with jumps

In this article, we establish a propagation of chaos result for weakly interacting nonlinear Snell envelopes which converge to a class of mean-field reflected backward stochastic differential equations (BSDEs) with jumps and right-continuous and left-limited obstacle, where the mean-field interaction in terms of the distribution of the Y-component of the solution enters both the driver and the lower obstacle. Under mild Lipschitz and integrability conditions on the coefficients, we prove existence and uniqueness of the solution to both the mean-field reflected BSDEs with jumps and the corresponding

system of weakly interacting particles and provide a propagation of chaos result for the whole solution (Y, Z, U, K), which requires new technical results due to the dependence of the obstacle on the solution and the presence of jumps.

[1] Djehiche, B., Dumitrescu, R., Zeng, J. *A propagation of chaos result for a class of mean-field reflected BSDEs with jumps.* arXiv preprint arXiv:2111.14315., 2021

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Mean-Field Doubly Reflected Backward Stochastic Differential Equations

We study mean-field doubly reflected BSDEs. Using the fixed point method, we show existence and uniqueness of the solution when the data which define the BSDE are p-integrable with $p = 1$ or $p > 1$. The two cases are treated separately. The penalization method is also considered.

[1] Chen, Y., Hamadene, S., Mu, T. *Mean-field doubly reflected backward stochastic differential equations.* arXiv preprint arXiv:2007.04598, 2020

Numerical methods for non-linear McKean-Vlasov SDEs

Organizers: Chaman Kumar and Gonçalo dos Reis

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Tamed Milstein-type scheme for the McKean-Vlasov SDEs

We present a tamed Milstein-type scheme for the interacting particle system associated with McKean–Vlasov SDEs using the notion of Lions derivative and investigate its convergence (with rate 1.0) in strong sense. The coefficients are assumed to have super-linear growth in the state variable and linear growth in the measure variable. We avoid the use of Ito's formula for functions depending on measures and require only the first order derivatives of the coefficients in both state and measure components. Our results are new even for the Lipschitz continuous coefficients. The talk is based on the joint work [1].

[1] Kumar, C., Neelima, Reisinger, C. and Stockinger W., *Well-posedness and tamed schemes for McKean–Vlasov Equations with Common Noise.* to appear in the *Annals of Applied Probability*, 2022

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Numerical approximation of McKean-Vlasov SDEs via SGD

We propose a novel approach of numerically approximate McKean-Vlasov SDEs that avoids altogether the usual interacting particle approximation and Propagation of Chaos results. After (possibly) leveraging the projection of SDEs coefficients on the Fourier basis [1], we use the stochastic gradient descent algorithm to approximate its measure-dependent components — our technique approximates the functional of the measure instead of the measure itself. This technique avoids particle approximation. We present several numerical experiments in support of the algorithm. This talk is based on joint work [2] with A. Agarwal (U. of Glasgow) and S. Pagliarani (U. of Bologna).

[1] Belomestny, D., and Schoenmakers, J. *Projected particle methods for solving McKean-Vlasov stochastic differential equations.* SIAM Journal on Numerical Analysis 56, 6, 3169–3195, 2018

[2] Agarwal, A., Pagliarani, S., and dos Reis, G. *Approximating McKean-Vlasov SDEs via SGD.* Forthcoming, 2022

Connections with PDEs - 3

Contributed Session
Chairwoman: Elena Issoglio

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Kolmogorov equations on spaces of measures associated to nonlinear filtering processes

The aim of this talk is to introduce a class of backward Kolmogorov equations on spaces of probability and positive measures, associated to measure-valued stochastic processes arising in the context of nonlinear filtering. Indeed, in the filtering framework one can formulate two stochastic differential equations, called Zakai and Kushner-Stratonovich equations, that are satisfied by a positive measure and a probability measure-valued process respectively. Thus, one can study the associated backward Kolmogorov equations, that are partial differential equations of parabolic type on the space of measures.

In literature, the Kolmogorov equations associated to nonlinear filtering processes have been studied assuming that the measure-valued processes admit a density and then by exploiting stochastic calculus techniques in Hilbert spaces. The approach used here differs from that one, since the existence of a density is not assumed and everything is done directly in the context of measures.

In the talk, we introduce tools that allow us to write the backward Kolmogorov equations on spaces of measures and then present an existence and uniqueness result for classical solutions. If it remains time, we will discuss also a well posedness result for viscosity solutions. In particular this is a preliminary step for the study of the Hamilton-Jacobi-Bellman equations associated to the optimal control problem with partial observation.

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Uniqueness of the filtering equations in the space of measures

In this work, we consider a general filtering problem of the following form

$$X_t = X_0 + \int_0^t f(s, X_s, Y_s)ds + \int_0^t \sigma_1(s, X_s, Y_s)dV_s + \int_0^t \sigma_2(s, X_s, Y_s)dW_s,$$
$$Y_t = Y_0 + \int_0^t h(s, X_s, Y_s)ds + \int_0^t k(s, Y_s)dW_s,$$

where only the process Y is observed, and one is interested in computing the evolution of the conditional law of X_t given $\{Y_s,\ 0 \le s \le t\}$. Here V and W are two mutually independent Brownian motions, all processes being multidimensional. The novelty in our work is that the matrix k is not assumed to be invertible, which forces us to assume that h is of the form

$$h(t,x,y) = h_1(t,y) + k(t,y)h_2(t,x,y)\,.$$

The same kind of filtering problem has been considered in [4], in the particular case where $\sigma_2 \equiv 0$ (signal and observation noises are independent), and the observation process Y is one dimensional.

Our main result is a uniqueness result for the Zakai equation, in a class of measure valued solutions. Note that we do not assume that either of the matrices $\sigma_1\sigma_1^*$ and $\sigma_2\sigma_2^*$ is non degenerate, and we hope to be able to treat the case of coefficients with linear growth (work in progress). Our uniqueness argument is a duality argument. Generalizing the results of [2] and [3], we prove that a certain system of BSPDEs, which is dual to a proper version of the Zakai equation, has a smooth enough solution. This approach generalizes the arguments in sections 4.2 and 4.5 of [1], which are based on duality with a system of backward deterministic PDEs, and treat respectively the classical filtering problem with independent and correlated signal and observation noises.

[1] A. Bensoussan, *Stochastic control of partially observable systems*. Cambridge University Press, Cambridge, 1992.

[2] K. Du, Q. Meng, A revisit of W_2^n-theory of super-parabolic backward stochastic partial differential equations in $\mathbb{R}^d$, *Stoch. Proc. and Applic.* **120**, 1996–2015, 2010.

[3] K. Du, S. Tang, Q. Zhang, $W^{m,p}$–solutions ($p \ge 2$) of linear degenerate backward stochastic partial differential equations in the whole space, *J. of Differential Equ.* **254**, 2877 –2904, 2013.

[4] D. J. Fotsa–Mbogne, E. Pardoux, Nonlinear filtering with degenerate noise, *Electron. Comm. Probab.* **22**, 44, pp. 1–14, 2017.

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Necessary and Sufficient Conditions for Optimal Control of Semilinear Stochastic Partial Differential Equations

In this talk, we consider the following optimal control problem of stochastic partial differential equations (SPDEs). Minimize

$$J(s,x;u) := \mathbb{E}\left[\int_s^T \int_\Lambda l(x_t^u(\lambda),u_t)\mathrm{d}\lambda\mathrm{d}t + \int_\Lambda h(x_T^u(\lambda))\mathrm{d}\lambda\right]$$

subject to

$$\begin{cases} \mathrm{d}x_t^u = [\Delta x_t^u + b(x_t^u,u_t)]\mathrm{d}t + \sigma(x_t^u,u_t)\mathrm{d}W_t, \quad t\in[s,T], \\ x_s^u = x \in L^2(\Lambda). \end{cases}$$

First we apply the spike variation method which relies on introducing the first and second order adjoint state. We give a novel characterization of the second order adjoint state as the solution to a backward SPDE on the space $L^2(\Lambda)\otimes L^2(\Lambda) \cong L^2(\Lambda^2)$. Using this representation, we prove the maximum principle for controlled SPDEs.

As another application of our characterization of the second order adjoint state, we derive additional necessary optimality conditions in terms of the value function. These results generalize a classical relationship between the adjoint states and the derivatives of the value function to the case of viscosity differentials.

The last part of the talk is devoted to sufficient optimality conditions. We show how the necessary conditions lead us directly to a non-smooth version of the classical verification theorem in the framework of viscosity solutions.

This talk is based on joint work with Wilhelm Stannat [W. Stannat, L. Wessels, *Peng's maximum principle for stochastic partial differential equations*, SIAM J. Control Optim., 59 (2021), pp. 3552–3573] and [W. Stannat, L. Wessels, *Necessary and Sufficient Conditions for Optimal Control of Semilinear Stochastic Partial Differential Equations*, https://arxiv.org/abs/2112.09639, 2022].

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