Multistage Financing Milestone Bonuses or Deferred Compensation

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Motivations

Common features of R&D projects and VC-backed startups:

- 1. Separation between technical experts and financiers leads to agency friction
- 2. Complex projects have multiple sequential stages
 - New drug development: lab testing, animal trials, and human trials
 - VC-backed startups have multiple financing rounds
- 3. Key personnel is important for success
 - Steven Jobs is important for early success of Apple
 - Agent has limited liability protection against outside value

Our setting

We consider a multistage project

- agent controls the intensity of stage completion and probability of success
- project terminates when agent's continuation value reaches outside value
- Baseline model: At the end of each stage, principal chooses between milestone cash bonus and deferred compensation

Extensions:

- 1. Principal observes a signal of agent's effort and can reward or penalize the agent according to signal observations
- 2. Principal can learn the type of project by exercising a costly real option

Our results

1. Pecking order between cash bonus and deferred compensation:

principal only pays cash bonus when deferred compensation reaches its maximum value

Reason: To maintain agent's skin in the project

Empirical connection: Ewens, Nanda, and Stanton (2020) show that cash compensation increases significantly for founder after the startup has a "product market fit"

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 Agent's equity stake is smaller in later stages than in early stage Reason: Uncertainty resolution increases project value in later stages Empirical connection: [Bengtsson and Sensory (2015), Ewens, Nanda, and Stanton (2020)]

Our results cont.

3. Monitoring and reward complement each other in the optimal contract

When positive (negative) evidence of the agent's effort accumulates, the principal increases (decreases) the reward and relax (intensifies) the monitoring

Reason: Reducing reward mitigates inefficient project termination

Empirical connection: Kaplan and Strömberg (2003) show that entrepreneurs have more cash flow rights under good performance and this state-contingencies are greater in the first VC rounds

Literature

Multistage projects and contact:

Green and Taylor (2006), Toxvaerd (2006), Varas (2018), Mayer (2021)

Monitoring and contact:

Piskorski and Westerfield (2016), Malenko (2019), Varas, Marinovic and Skrzypacz (2020), Chen, Sun, and Xiao (2020)

Model

Project:

- Project has N stages in sequence
- \blacktriangleright A lump sum payoff Δ at final success, zero payoff at failure of any stage
- No intermediate cash flow

Agent

- Risk neutral
- If the agent exerts effort a_t ∈ [0, ā], the current stage completes with probability a_tdt and it succeeds with probability

$$p^{(i)}\frac{a_t}{\bar{a}}$$
, in stage *i*.

- If the agent exerts a lower effort a ∈ [0, ā], he enjoys a private benefit λ(ā − a) per unit of time
- Agent's outside value is <u>U</u>, project is terminated when continuation value reaches <u>U</u>

Contract problem

- Risk neutral principal observes completion and success/failure of each stage
- Cumulative compensation C, nondecreasing
- If stage *i* is completed at ν

Success: new contact for the next stage: continuation utility $U_{\nu}^{(i+1)}$ and/or cash bonus: $R_{\nu}^{(i)}$

Failure: no reward

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Agent's problem:

$$\sup_{\boldsymbol{a}\in[0,\bar{a}]} \mathbb{E}^{\boldsymbol{a}} \Big[\int_{0}^{\tau\wedge\nu^{(1)}} e^{-\rho s} \big(dC_{s} + \lambda(\bar{a} - a_{s}) ds \big) \\ + \mathbb{I}_{\{\nu^{(1)}\leq\tau\}} e^{-\rho\nu^{(1)}} \boldsymbol{U}_{\nu^{(1)}}^{\boldsymbol{c}} + \mathbb{I}_{\{\nu^{(1)}>\tau\}} e^{-\rho\tau} \underline{\boldsymbol{U}} \Big],$$

 $u^{(1)}$ is the first stage completion time, au is the project termination time

$$U^{c}_{
u^{(1)}} = p^{(1)}(a_{
u}) \Big(U^{(2)}_{
u^{(1)}} + R^{(1)}_{
u^{(1)}} \Big)$$

is the expected continuation value right before first stage compltion ,

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Principal's problem

$$\begin{split} \sup_{U^{(i)},R^{(i)},C} \mathbb{E}^{\bar{a}} \Big[\int_{0}^{\tau \wedge \nu^{(1)}} e^{-rs} \big(-dC_{s} \big) + \mathbb{I}_{\{\nu^{(1)} \leq \tau\}} e^{-r\nu^{(1)}} V_{\nu^{(1)}}^{c} + \mathbb{I}_{\{\tau < \nu^{(1)}\}} e^{-r\tau} L \Big], \\ V_{\nu^{(1)}}^{c} &= p^{(1)} \Big(V_{\nu^{(1)}}^{(2)} - R_{\nu^{(1)}}^{(1)} \Big), \end{split}$$

$$V_{\nu^{(1)}}^{(2)} \text{ is the principal's value at the beginning of stage 2, subject to}$$

 \bar{a} is agent's optimal effort

In the final stage

$$V_{t}^{(N)} = \sup_{R^{(N)}, C} \mathbb{E}_{t}^{\bar{s}} \bigg[\int_{t}^{\nu^{(N)}} e^{-r(s-t)} (-dC_{s}) \\ + \mathbb{I}_{\{\nu^{(N)} \leq \tau\}} e^{-r(\nu^{(N)}-t)} p^{(N)} \bigg(\Delta - \mathcal{R}_{\nu^{(N)}}^{(N)} \bigg) + \mathbb{I}_{\{\tau < \nu^{(N)}\}} e^{-r(\tau-t)} \mathcal{L} \bigg].$$

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Agent's optimal effort

Agent's problem is transformed into

$$\mathbb{E}_{t}^{a}\left[\int_{t}^{\tau}\mathbb{I}_{\{\nu^{(i)}>s\}}e^{-\rho(s-t)}\left(dC_{s}+\lambda(\bar{a}-a_{s})ds\right)+\int_{t}^{\tau}\mathbb{I}_{\{\nu^{(i)}\in[s,s+ds)\}}e^{-\rho(s-t)}U_{s}^{c}ds$$
$$=\mathbb{E}_{t}^{a}\left[\int_{t}^{\tau}e^{-\int_{t}^{s}a_{u}du}e^{-\rho(s-t)}\left(dC_{s}+\lambda(\bar{a}-a_{s})ds+a_{s}U_{s}^{c}ds\right)+\cdots\right],$$

conditioning on $\{\nu^{(i)} > t\}$ in the first line

Lemma

The agent's continuation utility U follows the dynamics

$$dU_t = \rho U_t dt + \inf_{a_t \in [0,\bar{a}]} \left\{ a_t U_t - \lambda (\bar{a} - a_t) - a_t U_t^c \right\} dt - dC_t,$$

The agent's optimal effort is given by

$$a_t^* = \begin{cases} \bar{a}, & \underbrace{p^{(i)}(U_t^{(i+1)} + R_t^{(i)})}_{expected \ reward} \ge \underbrace{\lambda + U_t}_{expected \ cost}, \\ 0, & otherwise. \end{cases}$$

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Optimal contract: baseline case, single stage

Principal's HJB equation

$$(r+\bar{a})V = \sup_{R,c\geq 0}\left\{\bar{a}p(\Delta-R) + \left((\rho+\bar{a})U - \bar{a}pR\right)V' - (1+V')c\right\},\$$

with the boundary condition $V(\underline{U}) = L$

Define

$$\overline{U} = \inf\{U \ge \underline{U} : V' \le -1\}.$$

Proposition

- 0. When $r = \rho$, V admits a closed-form expression.
- 1. U decreases along path, project is terminated when U reaches \underline{U}
- 2. When the project succeeds, agent receives $R^* = \frac{1}{p}(\lambda + U)$
- 3. -V' < 1: marginal benefit of cash compensation is less than the marginal cost \Rightarrow no intermediate compensation

Optimal contract: baseline case, two stages

In the first stage, agent exerts \bar{a} if and only if

$$p^{(1)}(U_t^{(2)}+R_t^{(1)}) \geq \lambda+U_t.$$

Principal's HJB equation

$$(r+\bar{a})V^{(1)} = \max_{U^{(2)},R^{(1)}} \left\{ \bar{a}p^{(1)} \left(V^{(2)}(U^{(2)}) - R^{(1)} \right) + \left((\rho+\bar{a})U - \bar{a}p^{(1)}(U^{(2)} + R^{(1)}) \right) (V^{(1)})' \right\}$$

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First order conditions in $U^{(2)}$ and $R^{(1)}$:

$$(V^{(2)})'(U^{(2)}) - (V^{(1)})' = 0$$
 and $-1 - (V^{(1)})' = 0$

These two conditions cannot be satisfies simultaneously when $U^{(2)}$ is less than its maximum value, because

$$(V^{(2)})'(U^{(2)}) > -1$$

Main result

Proposition

- (i) Principal pays milestone bonus only when agent's continuation utility is sufficiently large before completion of the first stage
- (ii) When the first stage is successfully completed, the agent's continuation utility jumps up to start the second stage
- (iii) Agent's equity stake in the first stage is always larger than his equity stake in the second stage

$$\frac{U}{U+V^{(1)}(U)} > \frac{U}{U+V^{(2)}(U)}$$

(iii) is due to

$$\underbrace{p^{(1)}(V^{(2)}(U^{(2)}) + U^{(2)})}_{\text{1st stage expected payoff}} < \underbrace{p^{(2)}\Delta}_{\text{2nd stage expected payoff}}$$

A numeric example



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Extension: Monitoring

Principal observes a noisy signal y about the agent's effort

$$dy_t = a_t dt + \sigma dB_t^a,$$

Lemma

The agent's continuation utility U follows the dynamics

$$dU_t = \rho U_t dt + \inf_{a_t \in [0,\bar{a}]} \left\{ a_t U_t - \lambda (\bar{a} - a_t) - a_t U_t^c - \varphi_t a_t \right\} dt + \varphi_t dy_t - dC_t,$$

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ight.$$

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Single stage example



As $U \uparrow (y \uparrow)$

- Contract sensitivity $\downarrow 0$
- Milestone bonus $\uparrow \min\{(\lambda + \overline{U})/p, \Delta\}$

For multistage problems, main results for the baseline model still hold

Optimal signal precision

Principal can control $\sigma \in [\sigma_{\min}, \sigma_{\max}]$ Monitoring cost per unit of time

$$c(\sigma) = \frac{M}{\sigma^2}$$

Optimal signal precision

Principal can control $\sigma \in [\sigma_{\min}, \sigma_{\max}]$

Monitoring cost per unit of time

$$c(\sigma) = \frac{M}{\sigma^2}$$

Proposition

When R* is interior optimal, the optimal signal volatility is

$$\sigma^* = \begin{cases} \sigma_{\min}, & -\frac{\bar{a}^2(1+V')^2}{2V''} > M \\ \sigma_{\max}, & -\frac{\bar{a}^2(1+V')^2}{2V''} \le M \end{cases},$$
(1)

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An numeric example



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Conclusion

- A tractable multistage model
- Pecking order between cash bonus and deferred compensation
- Agent's equity stake is larger in early stage
- Monitoring and reward complement each other

Future work:

- Success depends on past actions
- Optimal design of stages

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Thanks for your attention!