ELECTRICITY DEMAND RESPONSE

A mean-field contract theory approach

Emma HUBERT

joint work with Romuald ÉLIE, Thibaut MASTROLIA, Dylan POSSAMAÏ

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1 ORFE Department, Princeton University
2 LAMA, Université Gustave Eiffel & Deepmind.
3 Berkeley IEOR Department
4 ETH Zurich, Department of Mathematics
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2. A principal – MF agents problem
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MOTIVATION & INTUITION
Supply-demand equilibrium for electricity required at all times, but inflexible (or at a high cost) production and random renewable energies.

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However: large variance in the consumer’s response to these mechanisms.
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3. How to take into account the large number of consumers?
► Goal of our contribution in Mean–field moral hazard for optimal energy demand response management (Mathematical Finance, 2021).

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**Asymmetries of information:**

**Moral Hazard:** the agent’s *behaviour* is not observable by the principal (second-best case).

The Agent (he) is a risk-averse consumer, who can deviate from his baseline consumption by reducing the mean and the volatility:

\[ X_t = x_0 - Z_t \alpha \cdot 1_{d} + Z_t \sigma(\beta) \cdot dW_s, \quad t \in [0,T], \quad (1) \]

where \( W \) is a \( d \)-dimensional Brownian Motion.

A control process for the agent is a pair \( \nu = (\alpha, \beta) \in U \):

- \( \alpha \) is the effort to reduce his consumption in mean;
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(i) identify a class of contracts, offered by the principal, that are revealing: the agent’s optimal response can be easily calculated;
(ii) prove that this restriction is without loss of generality, using 2BSDE;
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▶ The optimal form of contracts is as follows:

$$\xi_T = \xi_0 - \int_0^T H(X_s, \zeta_s) ds + \int_0^T Z_s dX_s + \frac{1}{2} \int_0^T \Gamma_s d\langle X \rangle_s + \frac{1}{2} R_A \int_0^T Z_s^2 d\langle X \rangle_s,$$

for an optimal choice of $\zeta = (Z, \Gamma)$ and $\xi_0$. 
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A PRINCIPAL – MF AGENTS PROBLEM
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- $\beta$, effort to reduce the volatility;
- $W$, d-dim. BM, representing the randomness specific to the agent;
- $W^o$, uni-dim. BM, representing the noise common to all agents.
Optimisation problem of the representative consumer:

\[
V^A_0(\xi) := \sup_{\nu=(\alpha,\beta)} \mathbb{E}^{IP} \left[ U_A \left( \xi - \int_0^T (c(\nu_t) - f(X_t)) \, dt \right) \right],
\]

where \( c \) is the cost of effort, \( f \) represents the agent’s preference towards his consumption, and \( U_A(x) = -e^{-R_A x} \).
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Aïd, Possamaï, and Touzi [1] (2019): Contract indexed on \( X \), and its quadratic variation \( \langle X \rangle \), through a process \((Z, \Gamma)\).

The principal chooses \((Z, \Gamma)\) in order to maximise her profit.
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Principal – multi-agents models: the principal can take advantage of the supplementary information available to her (see [4, 5]).
In our case, the principal can compute the distribution, \textit{conditional to common noise}, of the consumption of the others, denoted $\hat{\mu}$.

$\Rightarrow$ New form of contract: $\xi(X, \hat{\mu})$. 

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\xi_T = \xi_0 - \int_0^T \mathcal{H}(X_s, \zeta_s, \hat{\alpha}_s, \hat{\mu}_s) \, ds + \int_0^T Z_s \, dX_s + \frac{1}{2} \int_0^t (\Gamma_s + R_A \dot{Z}_s^2) \, d\langle X \rangle_s
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$$

$$
+ \int_0^T \mathbb{E}^{\hat{\mu}_s} \left[ Z_s^\mu (\hat{\chi}_s) \, d\hat{\chi}_s \right] + \int_0^T \tilde{f}(\hat{\mu}_s, Z_s, Z_s^\mu) \, ds,
$$

- $\hat{\alpha}^*$, the optimal effort of others on the drift of their consumption,
- $\hat{\chi}$, the consumption of others;
- $\mathbb{E}^{\hat{\mu}}$, expectation under $\hat{\mu}$ (with respect to the common noise);
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\xi_T &= \xi_0 - \int_0^T H(X_s, \zeta_s, \hat{\alpha}_s^*, \hat{\mu}_s)ds + \int_0^T Z_s dX_s + \frac{1}{2} \int_0^t (\Gamma_s + R_A Z^2_s) d\langle X \rangle_s \\
&\quad + \int_0^T \mathbb{E}_{\hat{\mu}_s} \left[ Z^\mu_s (\hat{X}_s) d\hat{X}_s \right] + \int_0^T \tilde{f}(\hat{\mu}_s, Z_s, Z^\mu_s) ds,
\end{align*}
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- $\hat{\alpha}^*$, the optimal effort of others on the drift of their consumption,
- $\hat{X}$, the consumption of others;
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$$+ \int_0^T \mathcal{E} \hat{\mu}_s \left[ \mathcal{Z}_s^\mu (\hat{\lambda}_s) \, d\hat{\lambda}_s \right] + \int_0^T \mathcal{F}(\hat{\mu}_s, Z_s, Z_s^\mu) \, ds,$$

• $\hat{\alpha}^*$, the optimal effort of others on the drift of their consumption,
• $\hat{\lambda}$, the consumption of others;
• $\hat{\mathcal{E}} \hat{\mu}$, expectation under $\hat{\mu}$ (with respect to the common noise);
• $\zeta_t = (Z_t, \Gamma, Z_t^\mu)$, parameters optimised by the principal.
• $\xi_0$, constant chosen by the principal in order to satisfy the participation constraint of the agent.
What is hidden behind this contract?

The contract is in fact indexed on:

- $X$, the deviation consumption of the representative consumer;
- $W^\circ$, the common noise.
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+ \int_0^T \sigma^\circ \bar{Z}_s^\mu \, dW_s^\circ + \frac{1}{2} R_A \int_0^T (\bar{Z}_s^\mu)^2 (\sigma^\circ)^2 \, ds + R_A \int_0^T Z_s \bar{Z}_s^\mu (\sigma^\circ)^2 \, ds,
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where \(\bar{Z}_t^\mu := \hat{\mathbb{E}}^\mu \left[ Z_t^\mu (\hat{X}_t) \right] \).
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+ \int_0^T \sigma^\circ \bar{Z}_s^\mu dW_s^\circ + \frac{1}{2} R_A \int_0^T (\bar{Z}_s^\mu)^2 (\sigma^\circ)^2 ds + R_A \int_0^T Z_s \bar{Z}_s^\mu (\sigma^\circ)^2 ds,
\]

where $\bar{Z}_t^\mu := \hat{R}_t^\mu [Z_t^\mu (\hat{X}_t)]$.

If the principal can offer contract depending directly on the common noise, she can offer this contract, indexed by $\bar{\zeta}_t = (\bar{Z}_t, \bar{Z}_t^\mu, \Gamma_t)$.
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The contract is in fact indexed on:

- \( X \), the deviation consumption of the representative consumer;
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+ \int_0^T \sigma^o \bar{Z}_s^\mu dW_s^o + \frac{1}{2} R_A \int_0^T \left( \bar{Z}_s^\mu \right)^2 (\sigma^o)^2 ds + R_A \int_0^T Z_s \bar{Z}_s^\mu (\sigma^o)^2 ds,
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where \( \bar{Z}_t^\mu := \hat{\mathbb{E}}^\mu [Z_t^\mu (\hat{X}_t)] \).

- If the principal can offer contract depending directly on the common noise, she can offer this contract, indexed by \( \bar{\xi}_T = (Z_t, \bar{Z}_t^\mu, \Gamma_t) \).
- Contracting on \( \hat{\mu} \) or \( W^o \) leads in fact to the same form of contract.
Equilibrium between agents: Given a contract of the previous form, indexed by $\zeta_t = (Z_t, \Gamma, Z_{t}^{\mu})$,

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**Principal’s problem:**

- this form of contract, where the principal chooses $\zeta := (Z, \Gamma, Z^{\mu})$, is **without loss of generality** $\iff$ second-order BSDE of the mean-field type;
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Principal’s problem:

- this form of contract, where the principal chooses $\zeta := (Z, \Gamma, Z^\mu)$, is without loss of generality $\Leftrightarrow$ second-order BSDE of the mean-field type;
- from the principal’s point of view, the contract $\xi$ is a function of $X$ and $\mu^X$, the conditional law of $X$. $\Leftrightarrow$ Problem of McKean-Vlasov type.
The principal wants to minimise, the sum of the conditional expectation of:

- the compensation $\xi$ paid to the consumers;
- the production cost of the consumption, $R_T \int_0^T g(X_t) dt$;
- the quadratic variation of the deviation consumption, $R_T \int_0^T d\langle X \rangle_t$.

With respect to the common noise.

Her problem is reduced to a standard control problem:

$$V_P := \sup_{\zeta \in V} E[h(U_P) - E[\mu_L T \mu_L] \xi_T + Z_T \int_0^T \langle X \rangle_t]$$

where $\mu_L$ is the conditional law of $L$ and $U_P(c) = -e^{-R_P c}$ or $U_P(c) = c$.
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$$V^P := \sup_{\zeta \in \mathcal{V}} \mathbb{E} \left[ U^P \left( - \mathbb{E}^{\mu^L_T}[L_T] \right) \right], \quad L_T = \xi_T + \int_0^T g(X_s)ds + \frac{h}{2} \int_0^T d\langle X \rangle_s,$$

where $\mu^L$ is the conditional law of $L$ and $U^P(c) = -e^{-R_c}$ or $U^P(c) = c$. 

Two state variables: the conditional law of $X$ ($\mu^X$) and the conditional law of $L$ ($\mu^L$) ⇒ HJB technics.
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- Two state variables: the conditional law of $X (\mu^X)$ and the conditional law of $L (\mu^L) \Rightarrow$ HJB technics.
Optimal indexation on the law

\[ Z^{\mu,*} = -Z^* + \frac{R_p}{R_A + R_p} \bar{u}^{p}_{\mu X}, \]

leads to the optimal contract:
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leads to the optimal contract:

\[ \xi_t = \xi_0 - \int_0^t \mathcal{H}(X_s, \mu_s^X, \zeta_s^*, \alpha_s^*) ds \]

\text{Hamiltonian}
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- Hamiltonian
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- **Hamiltonian**
- **Penalisation w.r.t the others**
- **Compensation for volatility control**

\[ + \frac{1}{2} \int_0^t \Gamma^*_s d\langle X\rangle_s \]
Optimal indexation on the law

\[ Z^{\mu,*} = -Z^* + \frac{R_P}{R_A + R_P} \bar{u}^p_{\mu^X}, \]

leads to the optimal contract:

\[ \xi_t = \xi_0 - \int_0^t \mathcal{H}(X_s, \mu^X_s, \zeta^*_s, \alpha^*_s) \, ds + \int_0^t Z^*_s (dX_s - \widetilde{E}_{\mu^X}^s \, d\tilde{X}_s) \]

\[ + \frac{1}{2} \int_0^t \Gamma^*_s \, d\langle X \rangle_s \]

Hamiltonian

Penalisation w.r.t the others

Compensation for volatility control

Payment on others

Compensation for risk due to the risk aversion of the consumer \((R_A)\)}
Optimal indexation on the law

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$$\xi_t = \xi_0 - \int_0^t \mathcal{H}(X_s, \mu^X_s, \zeta^*_s, \alpha^*_s) ds + \int_0^t Z^*_s (dX_s - \mathbb{E}^X_{\mu S} [d\tilde{X}_s])$$

- **Hamiltonian**
  $$+ \frac{1}{2} \int_0^t \Gamma^*_s d\langle X \rangle_s$$
- **Penalisation w.r.t the others**
  $$+ \frac{R_P}{R_A + R_P} \int_0^t \bar{u}^P_{\mu X} \mathbb{E}^X_{\mu S} [d\tilde{X}_s]$$
- **Compensation for volatility control**
  $$\frac{1}{2} R_A \int_0^t \left( (Z^*_s)^2 (d\langle X \rangle_s - (\sigma^o)^2 ds) + \frac{R_P^2}{(R_A + R_P)^2} (\sigma^o)^2 (\bar{u}^P_{\mu X})^2 ds \right).$$
- **Payment on others**
  $$\frac{1}{2} R_A \int_0^t \left( (Z^*_s)^2 (d\langle X \rangle_s - (\sigma^o)^2 ds) + \frac{R_P^2}{(R_A + R_P)^2} (\sigma^o)^2 (\bar{u}^P_{\mu X})^2 ds \right).$$
- **Compensation for risk due to the risk aversion of the consumer (R_A)**
Let $X^\circ$ be the consumption \textit{without common noise} (corrected for climatic hazards):

$$dX_t^\circ = -\alpha^*(Z_t^*)dt + \sigma^*(\Gamma_t^*) \cdot dW_t.$$
Let $X^\circ$ be the consumption **without common noise** (corrected for climatic hazards):

$$dX^\circ_t = -\alpha^*(Z_t^*)dt + \sigma^*(\Gamma_t^*) \cdot dW_t.$$ 

Rewriting of the contract: indexed on $X^\circ$ and $W^\circ$:

$$\xi_T = \xi_0 - \int_0^T \mathcal{H}(X_s, \zeta^*_s)ds + \int_0^T Z_s^* dX_s^\circ + \frac{1}{2} \int_0^T (\Gamma_s^* + R_A|Z_s^*|^2) d\langle X^\circ \rangle_s.$$
Let $X^\circ$ be the consumption *without common noise* (corrected for climatic hazards):

$$dX^\circ_t = -\alpha^*(Z^*_t)\,dt + \sigma^*(\Gamma^*_t) \cdot dW_t.$$ 

Rewriting of the contract: indexed on $X^\circ$ and $W^\circ$:

$$\xi_T = \xi_0 - \int_0^T \mathcal{H}(X_s, \zeta^*_s)\,ds + \int_0^T Z^*_s dX^\circ_s + \frac{1}{2} \int_0^T (\Gamma^*_s + R \cdot |Z^*_s|^2) d\langle X^\circ \rangle_s$$

$$+ R_p \sigma^\circ \int_0^T \bar{f}(s, \mu^X) dW^\circ_s$$
Let $X^o$ be the consumption without common noise (corrected for climatic hazards):

$$dX^o_t = -\alpha^*(Z^* t) dt + \sigma^*(\Gamma^* t) \cdot dW_t.$$ 

Rewriting of the contract: indexed on $X^o$ and $W^o$:

$$\xi_T = \xi_0 - \int_0^T \mathcal{H}(X_s, \zeta^*_s) ds + \int_0^T Z^*_s dX^o_s + \frac{1}{2} \int_0^T (\Gamma^*_s + R_A |Z^*|^2) d\langle X^o \rangle_s$$

$$+ R_P \sigma^o \int_0^T \tilde{f}(s, \mu^X) dW^o_s + \frac{1}{2} R_A R^2_P |\sigma^o|^2 \int_0^T |\tilde{f}(s, \mu^X)|^2 ds.$$
Let $X^o$ be the consumption without common noise (corrected for climatic hazards):

$$dX_t^o = -\alpha^*(Z_t^*)dt + \sigma^*(\Gamma_t^*) \cdot dW_t.$$ 

Rewriting of the contract: indexed on $X^o$ and $W^o$:

$$\xi_T = \xi_0 - \int_0^T \mathcal{H}(X_s, \zeta_s^*) ds + \int_0^T Z_s^* dX_s^o + \frac{1}{2} \int_0^T (\Gamma_s^* + R_A|Z_s^*|^2) d\langle X^o \rangle_s$$

$$+ R_P \sigma^o \int_0^T \bar{f}(s, \mu^X) dW^o_s + \frac{1}{2} R_A R_P^2 |\sigma^o|^2 \int_0^T |\bar{f}(s, \mu^X)|^2 ds.$$ 

Risk–neutral case ($R_P = 0$) $\Rightarrow$ Classic contract for drift and volatility control, indexed on $X^o$, the part of the deviation that is actually controlled by the agent.
NUMERICAL RESULTS
If the energy value discrepancy is linear, i.e. \((f - g)(x) = \delta x, x \in \mathbb{R}\):

- the optimal \(Z^*\) and \(\Gamma^*\) are deterministic functions of time;
- the payment \(Z^{\mu,*}\) allows the principal to choose the risk she wants to bear:

\[
Z^{\mu,*}_t = -Z^*_t + \frac{R_p}{R_A + R_p} \delta(T - t).
\]
If the energy value discrepancy is linear, i.e. \((f - g)(x) = \delta x, x \in \mathbb{R}\):

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Z_{t^*,*} = -Z_t^* + \frac{R_p}{R_A + R_p} \delta (T - t).
\]

We can compare the efforts and the utility of the principal when she offers contracts indexed by \(\zeta^0 = (Z, 0, \Gamma)\):

\[
\xi_T = \xi_0 - \int_0^T \mathcal{H}(X_s, \zeta^0_s) ds + \int_0^T Z_s dX_s + \frac{1}{2} \int_0^T \left( \Gamma_s + R_A Z_s^2 \right) d\langle X \rangle_s,
\]
Figure: Relative utility difference. Variation with respect to $R_P$ and $\sigma^\circ$. 

GAIN IN UTILITY FOR THE PRINCIPAL
Figure: Relative gain on efforts. Variation with respect to $R_P$ and $\sigma^\circ$. 
CONCLUSION
Technical contribution: Extension of PA problems with volatility control to a continuum of agents with mean-field interactions, by developing natural extensions of the 2BSDE theory.

- While the consumers are in a mean-field game...
- the principal faces a control problem of McKean Vlasov type.
Technical contribution: Extension of PA problems with volatility control to a continuum of agents with mean-field interactions, by developing natural extensions of the 2BSDE theory.

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Results: At the end, this more sophisticated form of contract:

- allows the principal to better share the risk induced by the common noise with the agent;
- provides better incentives to the agents.
Technical contribution: Extension of PA problems with volatility control to a continuum of agents with mean–field interactions, by developing natural extensions of the 2BSDE theory.

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Results: At the end, this more sophisticated form of contract:
▶ allows the principal to better share the risk induced by the common noise with the agent;
▶ provides better incentives to the agents.

Further works:
▶ more general model;
▶ application to finance, insurance...
BIBLIOGRAPHY


