ELECTRICITY DEMAND RESPONSE

A mean-field contract theory approach

Emma HUBERT[1]

joint work with Romuald ÉLIE[2] Thibaut MASTROLIA[3] Dylan POSSAMAÏ[4]

9th International Colloquium on BSDEs and Mean Field Systems

Annecy – June 27–July 1, 2022

[1]ORFE Department, Princeton University

[2]LAMA, Université Gustave Eiffel & Deepmind.

[3]Berkeley IEOR Department

[4]ETH Zurich, Department of Mathematics

MOTIVATION & INTUITION

- **Supply-demand equilibrium** for electricity required at all times, but **inflexible** (or at a high cost) production and **random** renewable energies.
- **Idea:** increase the flexibility of the demand, facilitated by the development of smart meters.

- **Supply-demand equilibrium** for electricity required at all times, but **inflexible** (or at a high cost) production and **random** renewable energies.
- **Idea:** increase the flexibility of the demand, facilitated by the development of smart meters.

1. How can we encourage demand management and reward it optimally?

- **In practice.** Tariff offers, price signals to encourage the consumers to reduce their consumption during peak demand periods.
- **However:** large variance in the consumer's response to these mechanisms.

- **Supply-demand equilibrium** for electricity required at all times, but **inflexible** (or at a high cost) production and **random** renewable energies.
- **Idea:** increase the flexibility of the demand, facilitated by the development of smart meters.

1. How can we encourage demand management and reward it optimally?

- **In practice.** Tariff offers, price signals to encourage the consumers to reduce their consumption during peak demand periods.
- **However:** large variance in the consumer's response to these mechanisms.

2. How to improve the responsiveness?

- Aïd, Possamaï, and Touzi [1] (2019) – Principal-agent model with volatility control, to improve the consumer's response.

- **Supply-demand equilibrium** for electricity required at all times, but **inflexible** (or at a high cost) production and **random** renewable energies.
- **Idea:** increase the flexibility of the demand, facilitated by the development of smart meters.

1. How can we encourage demand management and reward it optimally?

- **In practice.** Tariff offers, price signals to encourage the consumers to reduce their consumption during peak demand periods.
- **However:** large variance in the consumer's response to these mechanisms.

2. How to improve the responsiveness?

- Aïd, Possamaï, and Touzi [1] (2019) – Principal-agent model with volatility control, to improve the consumer's response.

3. How to take into account the large number of consumers?

- Goal of our contribution in Mean–field moral hazard for optimal energy demand response management (Mathematical Finance, 2021).

Noteworthy papers: Holmström and Milgrom [6] (1987), Sannikov [7] (2008).

- Analyse interactions between economic agents, in particular with asymmetric information.

Noteworthy papers: Holmström and Milgrom [6] (1987), Sannikov [7] (2008).

- Analyse interactions between economic agents, in particular with asymmetric information.

    The principal (she) initiates a contract for a period $[0, T]$.

    The agent (he) accepts or not the contract proposed by the principal.

Noteworthy papers: Holmström and Milgrom [6] (1987), Sannikov [7] (2008).

- Analyse interactions between economic agents, in particular with asymmetric information.

  The principal (she) initiates a contract for a period $[0, T]$.

  The agent (he) accepts or not the contract proposed by the principal.

The principal must suggest an optimal contract: maximises her utility, and that the agent will accept (reservation utility).

Noteworthy papers: Holmström and Milgrom [6] (1987), Sannikov [7] (2008).

- Analyse interactions between economic agents, in particular with asymmetric information.

  The principal (she) initiates a contract for a period $[0, T]$.

  The agent (he) accepts or not the contract proposed by the principal.

The principal must suggest an optimal contract: maximises her utility, and that the agent will accept (reservation utility).

Asymmetries of information:

Moral Hazard: the agent's behaviour is not observable by the principal (second-best case).

Aïd, Possamaï, and Touzi [1] (2019) - Principal–agent problem with moral hazard and volatility control.

Aïd, Possamaï, and Touzi [1] (2019) - Principal–agent problem with moral hazard and volatility control.

The Agent (he) is a risk-averse consumer, who can deviate from his baseline consumption by reducing the mean and the volatility:

Aïd, Possamaï, and Touzi [1] (2019) - Principal–agent problem with moral hazard and volatility control.

The Agent (he) is a risk-averse consumer, who can deviate from his baseline consumption by reducing the mean and the volatility:

$$X_t = x_0 - \int_0^t \alpha_s \cdot \mathbf{1}_d ds + \int_0^t \sigma(\beta_s) \cdot dW_s, \ t \in [0, T], \tag{1}$$

where W is a d–dimensional Brownian Motion.

Aïd, Possamaï, and Touzi [1] (2019) - Principal–agent problem with moral hazard and volatility control.

The Agent (he) is a risk-averse consumer, who can deviate from his baseline consumption by reducing the mean and the volatility:

$$X_t = x_0 - \int_0^t \alpha_s \cdot \mathbf{1}_d \mathrm{d}s + \int_0^t \sigma(\beta_s) \cdot \mathrm{d}W_s, \ t \in [0, T], \tag{1}$$

where W is a d–dimensional Brownian Motion.
A control process for the agent is a pair $\nu := (\alpha, \beta) \in \mathcal{U}$:

- α is the effort to reduce his consumption in mean;
- β is the effort to reduce the variability of his consumption.

Aïd, Possamaï, and Touzi [1] (2019) - Principal–agent problem with moral hazard and volatility control.

The Agent (he) is a risk-averse consumer, who can deviate from his baseline consumption by reducing the mean and the volatility:

$$X_t = x_0 - \int_0^t \alpha_s \cdot \mathbf{1}_d \mathrm{d}s + \int_0^t \sigma(\beta_s) \cdot \mathrm{d}W_s, \; t \in [0, T], \tag{1}$$

where W is a d–dimensional Brownian Motion.
A control process for the agent is a pair $\nu := (\alpha, \beta) \in \mathcal{U}$:

- α is the effort to reduce his consumption in mean;
- β is the effort to reduce the variability of his consumption.

The principal (she) is a producer (or a retailer) subject to energy generation costs and to consumption volatility costs.

The principal wants to incentivise the consumer to reduce the mean and the volatility of his consumption.

The principal wants to incentivise the consumer to reduce the mean and the volatility of his consumption.

Moral Hazard: She observes the consumption X of the agent in continuous time, but not the effort ν he makes.

The principal wants to incentivise the consumer to reduce the mean and the volatility of his consumption.

Moral Hazard: She observes the consumption X of the agent in continuous time, but not the effort ν he makes.

▶ Volatility control. Cvitanić, Possamaï, and Touzi [3] (2018)

(i) identify a class of contracts, offered by the principal, that are revealing: the agent's optimal response can be easily calculated;

(ii) prove that this restriction is without loss of generality, using 2BSDE;

(iii) solve the principal's problem, which is now standard.

The principal wants to incentivise the consumer to reduce the mean and the volatility of his consumption.

Moral Hazard: She observes the consumption X of the agent in continuous time, but not the effort ν he makes.

- Volatility control. Cvitanić, Possamaï, and Touzi [3] (2018)

 (i) identify a class of contracts, offered by the principal, that are revealing: the agent's optimal response can be easily calculated;
 (ii) prove that this restriction is without loss of generality, using 2BSDE;
 (iii) solve the principal's problem, which is now standard.

- The optimal form of contracts is as follows:

$$\xi_T = \xi_0 - \int_0^T \mathcal{H}(X_s, \zeta_s)ds + \int_0^T Z_s dX_s + \frac{1}{2}\int_0^T \Gamma_s d\langle X\rangle_s + \frac{1}{2}R_A \int_0^T Z_s^2 d\langle X\rangle_s,$$

for an optimal choice of $\zeta = (Z, \Gamma)$ and ξ_0.

The producer is facing a mean–field (MF) of correlated consumers and optimise in mean.

Find a way for the principal to benefit from dealing with this MF of consumers.

The producer is facing a mean–field (MF) of correlated consumers and optimise in mean.

Find a way for the principal to benefit from dealing with this MF of consumers.

She knows the law of the consumption of the pool of consumers.

- She can design a new contract in order to penalise / reward a consumer who makes less / more effort than the rest of the pool.

The producer is facing a mean–field (MF) of correlated consumers and optimise in mean.

Find a way for the principal to benefit from dealing with this MF of consumers.

She knows the law of the consumption of the pool of consumers.

▶ She can design a new contract in order to penalise / reward a consumer who makes less / more effort than the rest of the pool.

Intuition. Optimal contracts should consists of two parts:

- A classical part indexed on the deviation consumption of the agent (previous contract, as in [1]);

The producer is facing a mean–field (MF) of correlated consumers and optimise in mean.

Find a way for the principal to benefit from dealing with this MF of consumers.

She knows the law of the consumption of the pool of consumers.

- She can design a new contract in order to penalise / reward a consumer who makes less / more effort than the rest of the pool.

Intuition. Optimal contracts should consists of two parts:

- A classical part indexed on the deviation consumption of the agent (previous contract, as in [1]);
- An additional part indexed on the law of the deviation consumption of others.

The producer is facing a mean–field (MF) of correlated consumers and optimise in mean.

Find a way for the principal to benefit from dealing with this MF of consumers.

She knows the law of the consumption of the pool of consumers.

▶ She can design a new contract in order to penalise / reward a consumer who makes less / more effort than the rest of the pool.

Intuition. Optimal contracts should consists of two parts:

- A classical part indexed on the deviation consumption of the agent (previous contract, as in [1]);
- An additional part indexed on the law of the deviation consumption of others.

▶ **Contract theory with many agents:** see for example Élie and Possamaï [4] (2019), and Élie, Mastrolia, and Possamaï [5] (2018) for a continuum of agents.

A PRINCIPAL – MF AGENTS PROBLEM

Classic MFG framework: all agents are identical.

- Study of a 'normal' consumer, who has no impact on total consumption: the representative agent (he).

Classic MFG framework: all agents are identical.

- Study of a 'normal' consumer, who has no impact on total consumption: the representative agent (he).

- His consumption at time $t \in [0, T]$ is:

$$X_t = x_0 \tag{2}$$

where

Classic MFG framework: all agents are identical.

- Study of a 'normal' consumer, who has no impact on total consumption: the representative agent (he).
- His consumption at time $t \in [0, T]$ is:

$$X_t = x_0 - \int_0^t \alpha_s \cdot \mathbf{1}_d \mathrm{d}s + \int_0^t \sigma(\beta_s) \cdot \mathrm{d}W_s \tag{2}$$

where

- α, effort to reduce the mean of his consumption;
- β, effort to reduce the volatility ;
- W, d-dim. BM, representing the randomness specific to the agent;

Classic MFG framework: all agents are identical.

- Study of a 'normal' consumer, who has no impact on total consumption: the representative agent (he).
- His consumption at time $t \in [0, T]$ is:

$$X_t = x_0 - \int_0^t \alpha_s \cdot \mathbf{1}_d \mathrm{d}s + \int_0^t \sigma(\beta_s) \cdot \mathrm{d}W_s + \int_0^t \sigma^\circ \mathrm{d}W_s^\circ, \tag{2}$$

where

- α, effort to reduce the mean of his consumption;
- β, effort to reduce the volatility ;
- W, d-dim. BM, representing the randomness specific to the agent;
- W°, uni-dim. BM, representing the noise common to all agents.

- Optimisation problem of the representative consumer:

$$V_0^A(\xi) := \sup_{\nu=(\alpha,\beta)} \mathbb{E}^{\mathbb{P}}\left[U_A\left(\xi - \int_0^T \big(c(\nu_t) - f(X_t)\big)\mathrm{d}t\right)\right], \tag{3}$$

where c is the cost of effort, f represents the agent's preference towards his consumption, and $U_A(x) = -\mathrm{e}^{-R_A x}$.

- Optimisation problem of the representative consumer:

$$V_0^A(\xi) := \sup_{\nu=(\alpha,\beta)} \mathbb{E}^{\mathbb{P}}\left[U_A\left(\xi - \int_0^T \big(c(\nu_t) - f(X_t)\big)dt\right)\right], \tag{3}$$

where c is the cost of effort, f represents the agent's preference towards his consumption, and $U_A(x) = -e^{-R_A x}$.

- Aïd, Possamaï, and Touzi [1] (2019): Contract indexed on X, and its quadratic variation $\langle X \rangle$, through a process (Z, Γ).
- The principal chooses (Z, Γ) in order to maximise her profit.

▶ Optimisation problem of the representative consumer:

$$V_0^A(\xi) := \sup_{\nu=(\alpha,\beta)} \mathbb{E}^{\mathbb{P}}\left[U_A\left(\xi - \int_0^T (c(\nu_t) - f(X_t))\mathrm{d}t\right)\right], \tag{3}$$

where c is the cost of effort, f represents the agent's preference towards his consumption, and $U_A(x) = -e^{-R_A x}$.

▶ Aïd, Possamaï, and Touzi [1] (2019): Contract indexed on X, and its quadratic variation $\langle X \rangle$, through a process (Z, Γ).

▶ The principal chooses (Z, Γ) in order to maximise her profit.

▶ Principal – multi-agents models : the principal can take advantage of the supplementary information available to her (see [4, 5]).

- In our case, the principal can compute the distribution, conditional to common noise, of the consumption of the others, denoted $\hat{\mu}$.

$$\Rightarrow \text{New form of contract: } \xi(X, \hat{\mu}).$$

- In our case, the principal can compute the distribution, conditional to common noise, of the consumption of the others, denoted $\hat{\mu}$.

$$\Rightarrow \text{New form of contract: } \xi(X, \hat{\mu}).$$

- Using the 'chain rule with common noise' by Carmona and Delarue [2] (2018), 'revealing contracts' should be of the form:

- In our case, the principal can compute the distribution, conditional to common noise, of the consumption of the others, denoted $\hat{\mu}$.

$$\Rightarrow \text{New form of contract: } \xi(X, \hat{\mu}).$$

- Using the 'chain rule with common noise' by Carmona and Delarue [2] (2018), 'revealing contracts' should be of the form:

$$\xi_T = \xi_0 - \int_0^T \mathcal{H}(X_s, \zeta_s, \hat{\alpha}_s^\star, \hat{\mu}_s)\mathrm{d}s + \int_0^T Z_s \mathrm{d}X_s + \frac{1}{2}\int_0^t (\Gamma_s + R_A Z_s^2)\mathrm{d}\langle X \rangle_s$$

- In our case, the principal can compute the distribution, conditional to common noise, of the consumption of the others, denoted $\hat{\mu}$.

$$\Rightarrow \text{New form of contract: } \xi(X, \hat{\mu}).$$

- Using the 'chain rule with common noise' by Carmona and Delarue [2] (2018), 'revealing contracts' should be of the form:

$$\xi_T = \xi_0 - \int_0^T \mathcal{H}(X_s, \zeta_s, \hat{\alpha}_s^\star, \hat{\mu}_s)\mathrm{d}s + \int_0^T Z_s \mathrm{d}X_s + \frac{1}{2}\int_0^t (\Gamma_s + R_A Z_s^2)\mathrm{d}\langle X\rangle_s$$
$$+ \int_0^T \widehat{\mathbb{E}}^{\hat{\mu}_s}\left[Z_s^\mu(\widehat{X}_s)\mathrm{d}\widehat{X}_s\right] + \int_0^T \widetilde{f}(\widehat{\mu}_s, Z_s, Z_s^\mu)\mathrm{d}s,$$

- $\widehat{\alpha}^\star$, the optimal effort of others on the drift of their consumption,
- $\widehat{X}$, the consumption of others;
- $\widehat{\mathbb{E}}^{\hat{\mu}}$, expectation under $\hat{\mu}$ (with respect to the common noise);

▶ In our case, the principal can compute the distribution, conditional to common noise, of the consumption of the others, denoted $\hat{\mu}$.

$$\Rightarrow \text{New form of contract: } \xi(X, \hat{\mu}).$$

▶ Using the 'chain rule with common noise' by Carmona and Delarue [2] (2018), 'revealing contracts' should be of the form:

$$\xi_T = \xi_0 - \int_0^T \mathcal{H}(X_s, \zeta_s, \hat{\alpha}_s^\star, \hat{\mu}_s)\mathrm{d}s + \int_0^T Z_s \mathrm{d}X_s + \frac{1}{2}\int_0^t \big(\Gamma_s + R_A Z_s^2\big)\mathrm{d}\langle X\rangle_s$$
$$+ \int_0^T \widehat{\mathbb{E}}^{\hat{\mu}_s}\Big[Z_s^\mu(\widehat{X}_s)\mathrm{d}\widehat{X}_s\Big] + \int_0^T \widetilde{f}\big(\widehat{\mu}_s, Z_s, Z_s^\mu\big)\mathrm{d}s,$$

- $\widehat{\alpha}^\star$, the optimal effort of others on the drift of their consumption,
- $\widehat{X}$, the consumption of others;
- $\widehat{\mathbb{E}}^{\hat{\mu}}$, expectation under $\hat{\mu}$ (with respect to the common noise);
- $\zeta_t = \big(Z_t, \Gamma, Z_t^\mu\big)$, parameters optimised by the principal.

- In our case, the principal can compute the distribution, conditional to common noise, of the consumption of the others, denoted $\hat{\mu}$.

$$\Rightarrow \text{New form of contract: } \xi(X, \hat{\mu}).$$

- Using the 'chain rule with common noise' by Carmona and Delarue [2] (2018), 'revealing contracts' should be of the form:

$$\xi_T = \xi_0 - \int_0^T \mathcal{H}(X_s, \zeta_s, \hat{\alpha}_s^\star, \hat{\mu}_s)\mathrm{d}s + \int_0^T Z_s \mathrm{d}X_s + \frac{1}{2}\int_0^t (\Gamma_s + R_A Z_s^2)\mathrm{d}\langle X\rangle_s$$
$$+ \int_0^T \widehat{\mathbb{E}}^{\hat{\mu}_s}\left[Z_s^\mu(\widehat{X}_s)\mathrm{d}\widehat{X}_s\right] + \int_0^T \widetilde{f}(\widehat{\mu}_s, Z_s, Z_s^\mu)\mathrm{d}s,$$

- $\widehat{\alpha}^\star$, the optimal effort of others on the drift of their consumption,
- $\widehat{X}$, the consumption of others;
- $\widehat{\mathbb{E}}^{\hat{\mu}}$, expectation under $\hat{\mu}$ (with respect to the common noise);
- $\zeta_t = (Z_t, \Gamma, Z_t^\mu)$, parameters optimised by the principal.
- ξ_0, constant chosen by the principal in order to satisfy the participation constraint of the agent.

What is hidden behind this contract ?

The contract is in fact indexed on:

- X, the deviation consumption of the representative consumer;
- W°, the common noise.

What is hidden behind this contract ?

The contract is in fact indexed on:

- X, the deviation consumption of the representative consumer;
- W°, the common noise.

$$\xi_T = \xi_0 - \int_0^T \mathcal{H}(X_s, \zeta_s)ds + \int_0^T Z_s dX_s + \frac{1}{2}\int_0^T (\Gamma_s + R_A Z_s^2) d\langle X \rangle_s$$
$$+ \int_0^T \sigma^\circ \overline{Z}_s^\mu dW_s^\circ + \frac{1}{2} R_A \int_0^T (\overline{Z}_s^\mu)^2 (\sigma^\circ)^2 ds + R_A \int_0^T Z_s \overline{Z}_s^\mu (\sigma^\circ)^2 ds,$$

where $\overline{Z}_t^\mu := \widehat{\mathbb{E}}^{\hat{\mu}}\left[Z_t^\mu(\widehat{X}_t)\right]$.

What is hidden behind this contract ?

The contract is in fact indexed on:

- X, the deviation consumption of the representative consumer;
- W°, the common noise.

$$\xi_T = \xi_0 - \int_0^T \mathcal{H}(X_s, \zeta_s)\mathrm{d}s + \int_0^T Z_s \mathrm{d}X_s + \frac{1}{2}\int_0^T (\Gamma_s + R_A Z_s^2)\mathrm{d}\langle X\rangle_s$$
$$+ \int_0^T \sigma^\circ \overline{Z}_s^\mu \mathrm{d}W_s^\circ + \frac{1}{2} R_A \int_0^T \big(\overline{Z}_s^\mu\big)^2 \big(\sigma^\circ\big)^2 \mathrm{d}s + R_A \int_0^T Z_s \overline{Z}_s^\mu \big(\sigma^\circ\big)^2 \mathrm{d}s,$$

where $\overline{Z}_t^\mu := \widehat{\mathbb{E}}^{\hat{\mu}}\big[Z_t^\mu(\widehat{X}_t)\big]$.

▶ If the principal can offer contract depending directly on the common noise, she can offer this contract, indexed by $\overline{\zeta}_t = \big(Z_t, \overline{Z}_t^\mu, \Gamma_t\big)$.

What is hidden behind this contract ?

The contract is in fact indexed on:

- X, the deviation consumption of the representative consumer;
- W°, the common noise.

$$\xi_T = \xi_0 - \int_0^T \mathcal{H}(X_s, \zeta_s)\mathrm{d}s + \int_0^T Z_s \mathrm{d}X_s + \frac{1}{2}\int_0^T \big(\Gamma_s + R_A Z_s^2\big)\mathrm{d}\langle X\rangle_s$$
$$+ \int_0^T \sigma^\circ \overline{Z}_s^\mu \mathrm{d}W_s^\circ + \frac{1}{2} R_A \int_0^T \big(\overline{Z}_s^\mu\big)^2 \big(\sigma^\circ\big)^2 \mathrm{d}s + R_A \int_0^T Z_s \overline{Z}_s^\mu \big(\sigma^\circ\big)^2 \mathrm{d}s,$$

where $\overline{Z}_t^\mu := \widehat{\mathbb{E}}^{\hat\mu}\big[Z_t^\mu(\widehat{X}_t)\big]$.

▶ If the principal can offer contract depending directly on the common noise, she can offer this contract, indexed by $\overline{\zeta}_t = \big(Z_t, \overline{Z}_t^\mu, \Gamma_t\big)$.

▶ Contracting on $\hat\mu$ or W° leads in fact to the same form of contract.

Equilibrium between agents: Given a contract of the previous form, indexed by $\zeta_t = \left(Z_t, \Gamma, Z_t^\mu\right)$,

- the optimal effort of an agent depends only on Z;

Equilibrium between agents: Given a contract of the previous form, indexed by $\zeta_t = (Z_t, \Gamma, Z_t^\mu)$,

- the optimal effort of an agent depends only on Z;
- mean-field equilibrium: the optimal efforts are the same for all consumers, and thus $\widehat{X} \overset{\mathcal{L}}{\sim} X$ and $\hat{\mu} = \mu^X$;

Equilibrium between agents: Given a contract of the previous form, indexed by $\zeta_t = (Z_t, \Gamma, Z^\mu_t)$,

- the optimal effort of an agent depends only on Z;
- mean-field equilibrium: the optimal efforts are the same for all consumers, and thus $\widehat{X} \overset{\mathcal{L}}{\sim} X$ and $\hat{\mu} = \mu^X$;

Principal's problem:

- this form of contract, where the principal chooses $\zeta := (Z, \Gamma, Z^\mu)$, is **without loss of generality** $\Leftrightarrow$ second-order BSDE of the mean-field type;

Equilibrium between agents: Given a contract of the previous form, indexed by $\zeta_t = (Z_t, \Gamma, Z_t^\mu)$,

- the optimal effort of an agent depends only on Z;
- mean-field equilibrium: the optimal efforts are the same for all consumers, and thus $\widehat{X} \overset{\mathcal{L}}{\sim} X$ and $\hat{\mu} = \mu^X$;

Principal's problem:

- this form of contract, where the principal chooses $\zeta := (Z, \Gamma, Z^\mu)$, is **without loss of generality** $\Leftrightarrow$ second-order BSDE of the mean-field type;
- from the principal's point of view, the contract ξ is a function of X and μ^X, the conditional law of X. $\Leftrightarrow$ Problem of McKean-Vlasov type.

The principal wants to minimise, the sum of the conditional expectation of:

- the compensation ξ paid to the consumers;

The principal wants to minimise, the sum of the conditional expectation of:

- the compensation ξ paid to the consumers;
- the production cost of the consumption, $\int_0^T g(X_t)\mathrm{d}t$;

The principal wants to minimise, the sum of the conditional expectation of:

- the compensation ξ paid to the consumers;
- the production cost of the consumption, $\int_0^T g(X_t)\mathrm{d}t$;
- the quadratic variation of the deviation consumption, $\int_0^T \mathrm{d}\langle X\rangle_t$;

with respect to the common noise.

The principal wants to minimise, the sum of the conditional expectation of:

- the compensation ξ paid to the consumers;
- the production cost of the consumption, $\int_0^T g(X_t)\mathrm{d}t$;
- the quadratic variation of the deviation consumption, $\int_0^T \mathrm{d}\langle X\rangle_t$;

with respect to the common noise.

Her problem is reduced to a standard control problem:

$$V^P := \sup_{\zeta\in\mathcal{V}} \mathbb{E}\Big[U^P\big(-\mathbb{E}^{\mu_T^L}[L_T]\big)\Big], \quad L_T = \xi_T + \int_0^T g(X_s)\mathrm{d}s + \frac{h}{2}\int_0^T \mathrm{d}\langle X\rangle_s,$$

where μ^L is the conditional law of L and $U^P(c) = -\mathrm{e}^{-R_P c}$ or $U^P(c) = c$.

The principal wants to minimise, the sum of the conditional expectation of:

- the compensation ξ paid to the consumers;
- the production cost of the consumption, $\int_0^T g(X_t)dt$;
- the quadratic variation of the deviation consumption, $\int_0^T d\langle X\rangle_t$;

with respect to the common noise.

Her problem is reduced to a standard control problem:

$$V^P := \sup_{\zeta \in \mathcal{V}} \mathbb{E}\Big[U^P\big(-\mathbb{E}^{\mu^L_T}[L_T]\big)\Big], \quad L_T = \xi_T + \int_0^T g(X_s)ds + \frac{h}{2}\int_0^T d\langle X\rangle_s,$$

where μ^L is the conditional law of L and $U^P(c) = -e^{-R_P c}$ or $U^P(c) = c$.

▶ Two state variables: the conditional law of X (μ^X) and the conditional law of L (μ^L) $\Rightarrow$ HJB technics.

Optimal indexation on the law

$$Z^{\mu,\star} = -Z^{\star} + \frac{R_P}{R_A + R_P}\overline{u}^{P}_{\mu^X},$$

leads to the optimal contract:

Optimal indexation on the law

$$Z^{\mu,\star} = -Z^\star + \frac{R_P}{R_A + R_P}\overline{u}^P_{\mu^X},$$

leads to the optimal contract:

$$\xi_t = \xi_0$$

Optimal indexation on the law

$$Z^{\mu,\star} = -Z^\star + \frac{R_P}{R_A + R_P} \overline{u}^P_{\mu^X},$$

leads to the optimal contract:

$$\xi_t = \xi_0 - \underbrace{\int_0^t \mathcal{H}(X_s, \mu^X_s, \zeta^\star_s, \alpha^\star_s) ds}_{\text{Hamiltonian}}$$

Optimal indexation on the law

$$Z^{\mu,\star} = -Z^\star + \frac{R_P}{R_A + R_P}\overline{u}^P_{\mu^X},$$

leads to the optimal contract:

$$\xi_t = \xi_0 \underbrace{- \int_0^t \mathcal{H}(X_s, \mu_s^X, \zeta_s^\star, \alpha_s^\star)\mathrm{ds}}_{\text{Hamiltonian}} \underbrace{+ \int_0^t Z_s^\star \left(\mathrm{d}X_s - \widetilde{\mathbb{E}}^{\mu_s^X}\left[\mathrm{d}\widetilde{X}_s\right]\right)}_{\text{Penalisation w.r.t the others}}$$

Optimal indexation on the law

$$Z^{\mu,\star} = -Z^\star + \frac{R_P}{R_A + R_P}\overline{u}^P_{\mu^X},$$

leads to the optimal contract:

$$\xi_t = \xi_0 \underbrace{- \int_0^t \mathcal{H}(X_s, \mu^X_s, \zeta^\star_s, \alpha^\star_s)\mathrm{d}s}_{\text{Hamiltonian}} \underbrace{+ \int_0^t Z^\star_s \left(\mathrm{d}X_s - \widetilde{\mathbb{E}}^{\mu^X_s}\left[\mathrm{d}\widetilde{X}_s\right]\right)}_{\text{Penalisation w.r.t the others}}$$
$$\underbrace{+ \frac{1}{2}\int_0^t \Gamma^\star_s \mathrm{d}\langle X \rangle_s}_{\text{Compensation for volatility control}}$$

Optimal indexation on the law

$$Z^{\mu,\star} = -Z^\star + \frac{R_P}{R_A + R_P}\overline{u}^P_{\mu^X},$$

leads to the optimal contract:

$$\xi_t = \xi_0 \underbrace{- \int_0^t \mathcal{H}(X_s, \mu_s^X, \zeta_s^\star, \alpha_s^\star)\mathrm{d}s}_{\text{Hamiltonian}} \underbrace{+ \int_0^t Z_s^\star \left(\mathrm{d}X_s - \widetilde{\mathbb{E}}^{\mu_s^X}[\mathrm{d}\widetilde{X}_s]\right)}_{\text{Penalisation w.r.t the others}}$$

$$\underbrace{+ \frac{1}{2}\int_0^t \Gamma_s^\star \mathrm{d}\langle X\rangle_s}_{\text{Compensation for volatility control}} \quad \underbrace{+ \frac{R_P}{R_A + R_P}\int_0^t \overline{u}^P_{\mu^X}\widetilde{\mathbb{E}}^{\mu_s^X}[\mathrm{d}\widetilde{X}_s]}_{\text{Payment on others}}$$

Optimal indexation on the law

$$Z^{\mu,\star} = -Z^\star + \frac{R_P}{R_A + R_P}\overline{u}^P_{\mu^X},$$

leads to the optimal contract:

$$\xi_t = \xi_0 \underbrace{- \int_0^t \mathcal{H}(X_s, \mu^X_s, \zeta^\star_s, \alpha^\star_s)ds}_{\text{Hamiltonian}} \underbrace{+ \int_0^t Z^\star_s \left(dX_s - \widetilde{\mathbb{E}}^{\mu^X_s}\left[d\widetilde{X}_s\right]\right)}_{\text{Penalisation w.r.t the others}}$$

$$\underbrace{+ \frac{1}{2}\int_0^t \Gamma^\star_s d\langle X\rangle_s}_{\text{Compensation for volatility control}} \quad \underbrace{+ \frac{R_P}{R_A + R_P}\int_0^t \overline{u}^P_{\mu^X}\widetilde{\mathbb{E}}^{\mu^X_s}\left[d\widetilde{X}_s\right]}_{\text{Payment on others}}$$

$$\underbrace{+\frac{1}{2}R_A \int_0^t \left(\left(Z^\star_s\right)^2 \left(d\langle X\rangle_s - \left(\sigma^\circ\right)^2 ds\right) + \frac{R_P^2}{(R_A + R_P)^2}\left(\sigma^\circ\right)^2 \left(\overline{u}^P_{\mu^X}\right)^2 ds \right)}_{\text{Compensation for risk due to the risk aversion of the consumer } (R_A)}.$$

- Let X° be the consumption without common noise (corrected for climatic hazards) :

$$dX_t^\circ = -\alpha^\star(Z_t^\star)dt + \sigma^\star(\Gamma_t^\star) \cdot dW_t.$$

- Let X° be the consumption without common noise (corrected for climatic hazards) :

$$\mathrm{d}X_t^\circ = -\alpha^\star(Z_t^\star)\mathrm{d}t + \sigma^\star(\Gamma_t^\star)\cdot \mathrm{d}W_t.$$

- Rewriting of the contract: indexed on X° and W°:

$$\xi_T = \xi_0 - \int_0^T \mathcal{H}(X_s, \zeta_s^\star)\mathrm{d}s + \int_0^T Z_s^\star \mathrm{d}X_s^\circ + \frac{1}{2}\int_0^T \left(\Gamma_s^\star + R_A |Z_s^\star|^2\right)\mathrm{d}\langle X^\circ\rangle_s$$

- Let X° be the consumption without common noise (corrected for climatic hazards) :

$$dX_t^\circ = -\alpha^\star(Z_t^\star)dt + \sigma^\star(\Gamma_t^\star)\cdot dW_t.$$

- Rewriting of the contract: indexed on X° and W°:

$$\xi_T = \xi_0 - \int_0^T \mathcal{H}(X_s, \zeta_s^\star)ds + \int_0^T Z_s^\star dX_s^\circ + \frac{1}{2}\int_0^T \left(\Gamma_s^\star + R_A|Z_s^\star|^2\right)d\langle X^\circ\rangle_s$$
$$+ R_P\sigma^\circ \int_0^T \bar{f}(s, \mu^X)dW_s^\circ$$

- Let X° be the consumption without common noise (corrected for climatic hazards) :

$$dX^\circ_t = -\alpha^\star(Z^\star_t)dt + \sigma^\star(\Gamma^\star_t)\cdot dW_t.$$

- Rewriting of the contract: indexed on X° and W°:

$$\xi_T = \xi_0 - \int_0^T \mathcal{H}(X_s,\zeta^\star_s)ds + \int_0^T Z^\star_s dX^\circ_s + \frac{1}{2}\int_0^T \left(\Gamma^\star_s + R_A|Z^\star_s|^2\right)d\langle X^\circ\rangle_s$$
$$+ R_P\sigma^\circ \int_0^T \bar{f}(s,\mu^X)dW^\circ_s + \frac{1}{2}R_A R_P^2|\sigma^\circ|^2\int_0^T |\bar{f}(s,\mu^X)|^2 ds.$$

- Let X° be the consumption without common noise (corrected for climatic hazards) :

$$dX^\circ_t = -\alpha^\star(Z^\star_t)dt + \sigma^\star(\Gamma^\star_t) \cdot dW_t.$$

- Rewriting of the contract: indexed on X° and W°:

$$\xi_T = \xi_0 - \int_0^T \mathcal{H}(X_s, \zeta^\star_s)ds + \int_0^T Z^\star_s dX^\circ_s + \frac{1}{2}\int_0^T \left(\Gamma^\star_s + R_A |Z^\star_s|^2\right) d\langle X^\circ\rangle_s$$
$$+ R_P \sigma^\circ \int_0^T \bar{f}(s, \mu^X) dW^\circ_s + \frac{1}{2} R_A R_P^2 |\sigma^\circ|^2 \int_0^T |\bar{f}(s, \mu^X)|^2 ds.$$

- Risk–neutral case ($R_P = 0$) $\Rightarrow$ Classic contract for drift and volatility control, indexed on X°, the part of the deviation that is actually controlled by the agent.

NUMERICAL RESULTS

If the energy value discrepancy is linear, i.e. $(f-g)(x) = \delta x$, $x \in \mathbb{R}$:

- the optimal $Z^\star$ and $\Gamma^\star$ are deterministic functions of time;
- the payment $Z^{\mu,\star}$ allows the principal to choose the risk she wants to bear:

$$Z_t^{\mu,\star} = -Z_t^\star + \frac{R_P}{R_A + R_P}\delta(T-t).$$

If the energy value discrepancy is linear, i.e. $(f-g)(x) = \delta x$, $x \in \mathbb{R}$:

- the optimal $Z^\star$ and $\Gamma^\star$ are deterministic functions of time;
- the payment $Z^{\mu,\star}$ allows the principal to choose the risk she wants to bear:

$$Z_t^{\mu,\star} = -Z_t^\star + \frac{R_P}{R_A + R_P}\delta(T-t).$$

We can compare the efforts and the utility of the principal when she offers contracts indexed by $\zeta^0 = (Z, 0, \Gamma)$:

$$\xi_T = \xi_0 - \int_0^T \mathcal{H}(X_s, \zeta_s^0)\mathrm{d}s + \int_0^T Z_s \mathrm{d}X_s + \frac{1}{2}\int_0^T \left(\Gamma_s + R_A Z_s^2\right) \mathrm{d}\langle X\rangle_s,$$

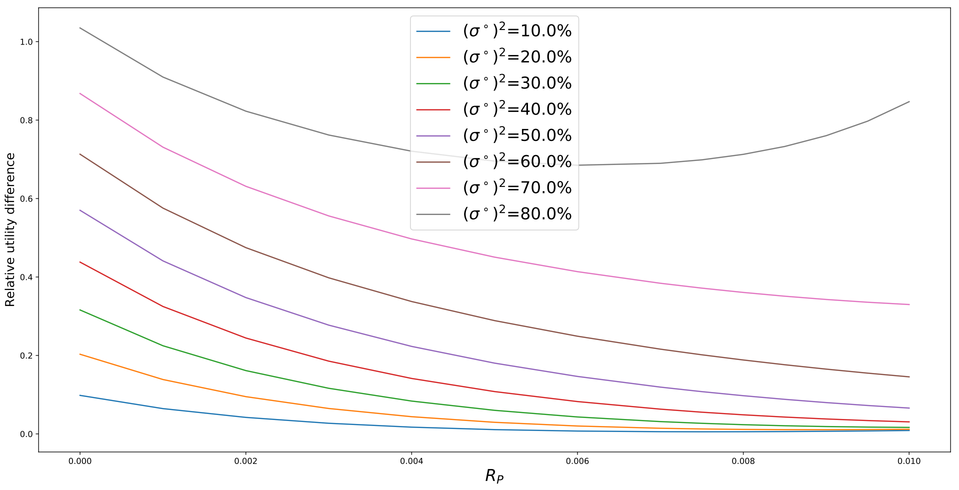

Figure: Relative utility difference.
Variation with respect to R_P and σ°.

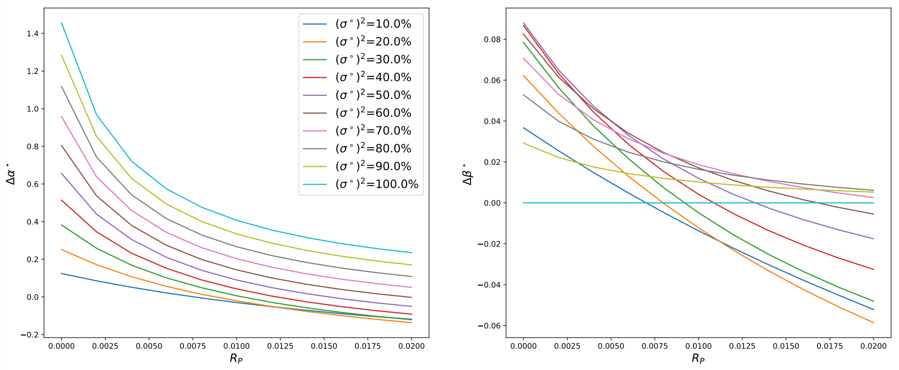

Figure: Relative gain on efforts.
Variation with respect to R_P and σ°.

CONCLUSION

Technical contribution: Extension of PA problems with volatility control to a continuum of agents with mean–field interactions, by developing natural extensions of the 2BSDE theory.

- While the consumers are in a mean-field game...
- the principal faces a control problem of McKean Vlasov type.

Technical contribution: Extension of PA problems with volatility control to a continuum of agents with mean–field interactions, by developing natural extensions of the 2BSDE theory.

- While the consumers are in a mean-field game...
- the principal faces a control problem of McKean Vlasov type.

Results: At the end, this more sophisticated form of contract:

- allows the principal to better share the risk induced by the common noise with the agent;
- provides better incentives to the agents.

Technical contribution: Extension of PA problems with volatility control to a continuum of agents with mean–field interactions, by developing natural extensions of the 2BSDE theory.

- While the consumers are in a mean-field game...
- the principal faces a control problem of McKean Vlasov type.

Results: At the end, this more sophisticated form of contract:

- allows the principal to better share the risk induced by the common noise with the agent;
- provides better incentives to the agents.

Further works:

- more general model;
- application to finance, insurance...

BIBLIOGRAPHY

[1] R. Aïd, D. Possamaï and N. Touzi. Optimal electricity demand response contracting with responsiveness incentives. Preprint arXiv:1810.09063, 2019.

[2] R. Carmona and F. Delarue. Probabilistic theory of mean field games with applications II, volume 84 of Probability theory and stochastic modelling. Springer International Publishing, 2018.

[3] J. Cvitanić, D. Possamaï and N. Touzi. Dynamic programming approach to principal–agent problems. Finance and Stochastics, 22(1):1–37, 2018.

[4] R. Élie and D. Possamaï. Contracting theory with competitive interacting agents. SIAM Journal on Control and Optimization, 57(2):1157–1188, 2019.

[5] R. Élie, T. Mastrolia and D. Possamaï. A tale of a principal and many, many agents. Mathematics of Operations Research, 44(2):440–467, 2018.

[6] B. Holmström and P. Milgrom. Aggregation and linearity in the provision of intertemporal incentives. Econometrica, 55(2):303–328, 1987.

[7] Y. Sannikov. A continuous–time version of the principal–agent problem. The Review of Economic Studies, 75(3):957–984, 2008.